Precalculus

Name

- Unit 2 Worksheet 1
 - 1. The relation described by the set of points $\{(-2,5), (0,5), (3,8), (3,9)\}$ is NOT a function. Explain why.

For Questions 2-4, use the graph at the right.

- 2. Explain why this graph represents a function.
- 3. Where is the function discontinuous? Describe each type of discontinuity.



4. Using interval notation, describe the domain and range of the function above.

5. What are the 3 domain issues you must remember in this course?



6. Graph each of the following functions. What do you notice? What happens when x = 2 on the graph of *b*?

7. What is the domain and range of the two functions above?

8. For each of the following functions, describe in interval notation and then, for any values NOT in the domain, identify the type of discontinuity (if they exist). You need to find the domain without using your calculator. You MAY use your calculator to determine the type of discontinuity, but we will find the discontinuity algebraically later.

a)
$$f(x) = x^3 - 2x^2 + 1$$

Domain:

Discontinuity:

b) $g(x) = \log_3(x - 4)$

Domain:

Discontinuity:

c)
$$h(x) = \frac{\sqrt{x+4}}{x-3}$$

Domain:

Discontinuity:

e)
$$z(x) = \frac{8x^2 - 14x - 15}{6x^2 - 13x - 5}$$

Domain:

Discontinuity:

g)
$$j(x) = \frac{1}{x} + \frac{5}{x-2}$$

d)
$$k(x) = \frac{x^2 - 2x}{x}$$

Domain:

Discontinuity:

f)
$$p(x) = \frac{3x^2 + 5x + 2}{(x+1)\sqrt{2x+9}}$$

Domain:

Discontinuity:

$$j(x) = \frac{1}{x} + \frac{5}{x-2}$$
h) $q(x) = \frac{\sqrt{4-x^2}}{x-3}$
Domain:
Discontinuity:
Discontinuity:

The given function is only drawn for $x \ge 0$. Complete the function for x < 0 with the following conditions: 9.



$$(x) = \frac{\sqrt{4-x^2}}{x-3}$$

Discontinuity:

- 10. Suppose you know the point (-2, -10) is on the graph of a function.
 - a. If the function is ODD, what other point is on the function?
 - b. If the function is EVEN, what other point is on the function?
- 11. Use the graph to the right to answer the following questions.
 - a. Identify all extrema.
 - b. Identify all the intervals on which the function is increasing and decreasing.





b. Identify all the intervals on which the function is increasing and decreasing.

12. Use the graph to the right to answer the following questions.

- 13. Use your graphing calculator to graph the function $g(x) = -x^3 + 3x 3$.
 - a. Identify all extrema.

a. Identify all extrema.

- b. Identify the intervals on which the function is increasing and decreasing.
- 14. Determine whether the following functions are bounded above, bounded below, bounded, or not bounded.

a)
$$y = 32$$
 b) $y = 2^x$ c) $y = 2 - x^2$ d) $y = \sqrt{1 - x^2}$ e) $y = x^2 \sqrt{x + 4}$

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Unit 2 -	Worksheet 2						
1.	Let $f(x) = -2x^3$	$^{3} + 7x$ and $g(x)$	= 2x - 5	5. Answer the follow	ving.		
a)	-f(x)	b)	f(-x)	C	c) $g(-a)$	d)	-g(a)

2. How do you algebraically prove that a function is ODD or EVEN?

3. Prove whether each function is even, odd, or neither. SHOW ALL STEPS!!

a)
$$f(x) = \sqrt{x^2 + 2}$$

b) $g(x) = 2x^3 - 3x$
c) $h(x) = -x^2 + 0.03x + 5$

d)
$$k(x) = x^3 + 4.2x^2 - 7$$
 e) $g(x) = \frac{3x^4}{1+x^2}$ f) $j(x) = \frac{5}{|x|}$

- 4. Write the end behavior of the function using limit notation. Graphing calculator is allowed.
- a) $f(x) = -x^3 2$ b) $f(x) = xe^{-x}$ c) f(x) = |3 - x|

d)
$$f(x) = \frac{-x^3}{x-1}$$
 e) $f(x) = \frac{4x-1}{x+3}$ f) $f(x) = \frac{4x}{x^2+1}$

5. The last three functions above are called *Rational Functions*.

- a. Explain why these functions are named so.
- b. What do you notice about the end behavior of #4d as compared to the other rational functions?

- 6. For a rational function, how can you tell whether a discontinuity is a hole or a vertical asymptote?
- 7. For each function below, find the following WITHOUT A CALCULATOR:
- i) Domain ii) Vertical Asymptotes or Holes a) $f(x) = \frac{2x-1}{3x+5}$ b) $g(x) = \frac{(3x-5)(x-8)}{x^2-4}$ c) $h(x) = \frac{2x-9}{x^2-x-6}$

d)
$$k(x) = \frac{2x+4}{x^2-3x-10}$$
 e) $p(x) = \frac{x+2}{3-x}$ f) $q(x) = \frac{4}{x^2+9}$

- 8. Sketch a freehand graph of a function with domain $(-\infty, 0) \cup (0, \infty)$ that satisfies ALL of the listed conditions.
 - a. f has a non-removable discontinuity at x = 0 and vertical asymptote at x = 0
 - b. *f* has relative maximum of -3 at x = 5 and an absolute minimum of -5 at x = -2
 - c. f has a removable discontinuity at x = 7



- 9. Sketch a freehand graph of a function with domain $(-\infty, \infty)$ that satisfies ALL of the listed conditions.
 - a. *f* is continuous for all *x*
 - b. f(-x) = f(x)
 - c. f is increasing on [0.2) and decreasing on $[2, \infty)$
 - d. f(2) = 3



Precalculus

Unit 2 - Worksheet 3

Name

1. Sketch the 12 basic functions from memory

$$y = x$$
 $y = x^2$ $y = x^3$ $y = \sqrt{x}$ $y = \frac{1}{x}$

$$y = b^x$$
 $y = \log_b x$ $y = \sin x$ $y = \cos x$ $y = |x|$

$$y = \lfloor x \rfloor \qquad \qquad y = \frac{1}{1 - e^{-x}}$$

Use the equation(s) above (not the names) to answer questions 2-15.

- 2. Seven of the twelve basic functions have the property of f(0) = 0. Which five do not?
- 3. Identify the four basic functions that are odd.
- 4. How many of the twelve basic functions are even? List them.
- 5. Identify the six basic functions that are increasing on their entire domains.
- 6. Identify the three basic functions that are decreasing on the interval $(-\infty, 0)$.
- 7. Only three of the twelve basic functions are bounded. Which three?
- 8. Which of the twelve basic functions are not continuous? Identify the types of discontinuity in each function.
- 9. Identify the three basic functions with no zeros.
- 10. How many of the twelve basic functions have a range of all real numbers? List them.
- 11. Identify the four functions that do NOT have end behavior $\lim_{x \to \infty} f(x) = \infty$.
- 12. How many of the twelve basic functions have end behavior $\lim_{x \to -\infty} f(x) = -\infty$? List them.
- 13. How many of the twelve basic functions look the same when flipped about the *y*-axis? List them.
- 14. How many of the twelve basic functions look the same upside down as right-side up? List them.
- 15. How many of the twelve basic functions are bounded below? List them.

16. Each of the ten graphs below is a slight variation (a transformation) of one of the parent functions. Match the graph to the correct equation.



y = [x + 1]

х

$$y = 2 - \frac{4}{1 + e^{-x}}$$

y x

х

х

X

- 17. Graph each of the following functions below on your calculator, then answer questions *i* and *ii*:
 - i) How does the graph relate to a graph of one of the twelve basic functions?
 - ii) Identify any extrema, if they exist.
- a) $r(x) = \sqrt{x 10}$ b) $q(x) = e^x + 2$ c) h(x) = |x| 10

18. The graph of $f(x) = \sqrt{x^2}$ is one of the twelve basic functions. Guess which one, then graph f(x) on your calculator. Were you right? If not, which of the basic twelve functions is f(x)?

Piecewise Functions

Piecewise Functions are simply functions that have been broken into 2 or more "pieces", where each piece is a portion of the graph with a limited domain. The limitations on the domain allow for the overall equation to pass the vertical line test, and thus be called a function.

19. The function y = |x| can be written as a piecewise function. Draw the graph of y = |x|, and then fill in the blanks below with the appropriate domain for each piece to complete the piecewise function of y = |x|.





20. Write a piecewise function for the graph below.





22. At a recent softball game Bill hit a double. He ran from home to second base (a distance of 120 feet) at a speed of 20 feet/sec. The next batter was up to bat for 1 minute but popped up to the pitcher, so Bill did not go anywhere. The third batter hit the first pitch over the fence for a homerun, so Bill was able to jog home (a distance of 120 feet) at a speed of 10 feet/sec. Write a piecewise function for the distance Bill traveled as a function of time in seconds.

23. An earthquake that occurred at 9:17 AM cracked a water tower in a small town. Water began leaking out of the tower at a rate of 12 cm³/min for the first 30 minutes. The rate then increased to 25 cm³/min for the next 40 minutes before the leak was fixed. Write a piecewise function for the amount of water W that leaked out of the tower as a function of time t. (What time should you let t = 0 represent? _____)

- 24. Two functions are said to be an inverse of each other (more on this later) if the graph of one can be obtained from the other by reflecting it across the line y = x.
 - a. Two of the twelve basic functions in this section are inverses of each other. Which are they?
 - b. Two of the twelve basic functions in this section are their own inverses. Which are they?
 - c. If you restrict the domain of one of the twelve basic functions to [0,∞), it becomes the inverse of another one.
 Which are they?

Precalculus

Unit 2 - Worksheet 4

For Questions 1-8, find the following for each function:

a) Identify the parent function and describe the transformation

- b) Accurately graph the function without you calculator
- c) Identify the domain and range
- 1. $y = 3 \log_3(x + 5)$









a) _____ b) For Questions 10 and 11, transform the given function by... a) ...a vertical stretch by a factor of 2 b) ... a reflection over the y-axis c) ... a horizontal shrink by a factor of $\frac{1}{2}$ d) ... a reflection over the *x*-axis 10. $f(x) = x^3 - 4x$ a) _____ c) _____ b) _____ d) _____ 11. $f(x) = \frac{1}{x+2}$ c) _____ a) _____ b) _____ d) _

Multiple Choice: For questions 12 and 13, suppose (6, 2) is a point on the graph of f(x).

12. Which point must be on the graph of f(-x)?A) (2,6)B) (6,-2)C) (-6,2)D) (-2,6)13. Which point must be on the graph of $\frac{1}{2}f(2x)$?A) (12,4)B) (3,1)C) (12,1)D) (3,4)

For Questions 14 and 15, use the graphs of f(x) shown below to graph each transformation.

14. -f(2x-6)





For Questions 16 and 17, use the given graph to write the equation of the function shown. Remember to consider the parent function and the transformations that were applied.

17.

16.



Based on your knowledge of parent functions, use each given data set to identify which parent function fits the data. Explain your choice by using the vocabulary from the first lesson.



Name

YOU HAVE 2 BLOCKS TO COMPLETE THIS WORKSHEET - DUE_

Adding, Subtracting, Multiplying, and Dividing Functions

- 1. Suppose f(x) = 2x + 1 and $g(x) = x^2$.
 - a. Find (f + g)(x) and state the domain.
 - b. Find (g f)(x) and state the domain.
 - c. Find (fg)(x) and state the domain.
- 2. Find $\left(\frac{f}{g}\right)(x)$ and $\left(\frac{g}{f}\right)(x)$ and state the domain for each of the following functions.
 - a. $f(x) = \sqrt{x+3}$ and $g(x) = x^2$ b. $f(x) = \sqrt{x-2}$ and $g(x) = \sqrt{x+4}$

Composite Functions

3. Find h(k(x)) and k(h(x)) for the following functions and state the domain for each.

a.
$$h(x) = x^2 - 3x + 1$$
 and $k(x) = \log_5(x)$
b. $h(x) = \frac{3x}{x+7}$ and $k(x) = e^x$

- 4. You don't always have to create a new function in order to evaluate the function. If I write h(k(9)) it means to plug 9 into the function k, then plug the result into h.
 - a. Suppose $m(x) = x^2 2x + 3$ and n(x) = 10x + 3. Evaluate n(m(-2)).
 - b. Suppose $p(x) = \sqrt{2x + 10}$ and $q(x) = \log_2(x)$. Evaluate p(q(8)).

In calculus, there is a rule called the chain rule (it is used to take a derivative of composite functions). In order to apply this rule, you must determine which component of the function is the "inside" and which is "outside". This skill is equivalent to finding the two functions that make up a composite function f(g(x)), where g(x) is the "inside" function.

5. Find f(x) and g(x) so that each function can be described as y = f(g(x)). a. $y = \sqrt{x^2 + 5x}$ b. y = |3x - 2| c. $y = e^{\sin x}$

6. If f and g are odd functions, SHOW that the composite function $f \circ g$ is also odd.

- 7. The surface area S (in square meters) of a hot-air balloon is given by $S(r) = 4\pi r^2$, where r is the radius of the balloon (in meters). If the radius is increasing with time t (in seconds) according to the formula $r(t) = \frac{2}{3}t^3$, $t \ge 0$, find the surface area of the balloon function of the time t.
- 8. The volume V of a right circular cylinder is $V = \pi r^2 h$. If the radius is half the height, express the volume V as a function of h.

- 9. The volume V of a right circular cone is $V = \frac{1}{3}\pi r^2 h$. If the radius is one-third of the height, express the volume V as a function of r.
- 10. The price p of a certain product and the quantity x sold obey the demand equation $p = -\frac{1}{4}x + 100, 0 \le x < 400$. Suppose that the cost C of producing x units is $C = \frac{\sqrt{x}}{25} + 600$. Assuming that all items produced are sold, find the cost C as a function of the price p.

- 11. What does the Vertical Line Test tell you about a graph? What does the Horizontal Line Test tell you about a graph?
- 12. For each relation graphed below, determine whether or not the relation represents a function AND whether or not the inverse of the relation represents a function.





13. Which ordered pair is in the inverse of the relation given by $x^2y + 5y = 9$? a. (2,1) b. (-2,1) c. (-1,2) d. (2,-1) e. (1,-2)

Finding and Verifying Inverses

14. How do you find an inversed of a function algebraically?

15. How do you prove that two functions are inverses of each other?

16. Find a formula for the inverse function AND prove that the two functions are inverses of each other. a. h(x) = 2x + 5b. $g(x) = \sqrt[3]{x+5} - 2$ 17. Find a formula for the inverse function AND prove that the two functions are inverse of each other if $f(x) = \frac{3x+1}{x-4}$.

- 18. A teacher gives a challenging algebra test. The lowest score is 52 which the teacher decides to scale to 70. The highest score is 88, which the teacher decides to scal to 97.
 - a. Using the points (50,70) and (88,97) find a linear equation that can be used to covert raw scores (original) to scaled scores (new).
 - b. Find the inverse of the function defined by this linear equation. What does the inverse function do?

19. To covert from x degrees Celsius to y degrees Fahrenheit, we use the formula $y = f(x) = \frac{9}{5}x + 32$. To covert from x degrees Farenheit, to y degrees Celsius, we use the formula $y = g(x) = \frac{5}{9}(x - 32)$. Show that f and g are inverses.

Precalculus Unit 2 - Worksheet 6

In order to model a real problem situation with a function, the first step is to define your variables. One useful strategy for problems that "appear" to have too many variables is to define one variable "in terms of" another variable. This should leave only the independent and depended variables remaining. Remember, sketches may be helpful as well.

- 1. If the length of a rectangle is 1 meter more than three times the width, then write an equation for the perimeter of the rectangle in terms of width of the rectangle.
- 2. The height of a right circular cylinder equals its diameter. Write the volume of the cylinder as a function of its radius.
- 3. One leg of a right triangle is twice as long as the other. Write the length of the hypotenuse as a function of the shorter leg.
- 4. The base of an isosceles triangle is half as long as the two equal sides. Write the area of the triangle as a function of the length of the base.

Once the equation is set up, with enough additional information, you can solve the problem.

- 5. Bob installed new windows in his house which were advertised to cut his energy bill by 7.1% per month after installation. If his new energy bill was \$315, what was his monthly bill before he installed the new windows?
- 6. When a number is added to its double and its triple, the sum is 714. Find the three numbers.
- 7. Ruth is weighing two job offers from the sales departments at two competing companies. One offers a base salary of \$35,000 plus a commission of 5% of the gross sales; the other company offers a base salary of \$28,000 plus 7% commission. What would Ruth's gross sales need to be in order for the second job offer to be more appealing than the first?
- 8. Emily's Decorating Service recommends putting a border around the top of four walls in a dining room that is 3 feet longer than it is wide. Find the dimensions of the room if the border needed is 54 feet.

You are now ready to tackle the "mixture" type problems. Remember to define your variables!

- 9. Coffee worth \$3.75 a pound was mixed with coffee worth \$4.35 a pound to produce a blend worth \$4.11 a pound. How much of each kind of coffee was used to produce 40 pounds of the blended coffee?
- 10. A 3% solution of sulfuric acid was mixed with an 18% solution of sulfuric acid to produce an 8% solution. How much 3% solution and how much 18% were used to produce 15 L of the 8% solution?
- 11. Reggie invests \$12,000, part at 7% annual interest and the rest at 8.5% annual interest. How much money was invested at each rate if Reggie's total annual interest was \$900?
- 12. Josh's Pre-Calculus final exam is 20% of his overall semester grade. Josh didn't study and the grade on his final exam was 13.5 points below his current grade. If Josh's overall semester grade ended up to be 88.525%, what was his current grade going into the final exam? What did he score on the final exam? Do you think Josh should have studied?

The final concept in modeling with functions is that one function can be used to determine the maximum or minimum for a certain variable.

- 13. Suppose a piece of cardboard 100 cm by 80 cm is used to make an open box by cutting out squares that are x cm by x cm from the corners.
 - a. Express the volume of the box as a function of length *x* of the square that is cut out of each corner.
 - b. What is the domain of this function?
 - c. Find the maximum volume of the box that can be made. What are the dimensions of this box and what is the length of *x* that was cut out of the original cardboard?

- 14. A 216 square meter rectangular pea patch is to be enclosed by a fence and divided into two equal parts by another fence parallel to the shorter sides.
 - a. What dimensions for the outer rectangle will require the smallest total length of fence?

b. How much fence is needed?

Precalculus Unit 2 - Review

NO CALCULATOR ALLOWED

For Questions 1-3 do the following:

- a) Identify the parent function
- c) Identify the domain and range

1.
$$y = 4^{2x} - 5$$



b) Describe the transformation

d) Accurately graph each function



3.
$$y = \frac{1}{x+2} + 3$$



5. Find the domain of each function.

a)
$$g(x) = \sqrt{9 - 7x}$$

4. $f(x) = \begin{cases} (x-2)^2 + 3 & \text{if } x \le 1 \\ 2|x| & \text{if } x > 1 \end{cases}$ y



▶ x



For Questions 6 & 7, a) Find the domain... and b) Determine if there is(are) any vertical asymptote(s) and/or holes.

6.
$$g(x) = \frac{4x^2+6}{x+1}$$
 7. $h(x) = \frac{5x^2+2}{x^2-9}$

8. Describe the end behavior of the function below using limit notation.



9. A thermometer is removed from a cup of coffee and placed in water with a constant temperature. The temperature was recorded at various times over the next 30 seconds. The results are graphed below.

What parent function seems to fit the data best?

Use vocabulary from this chapter to justify your response.



GRAPHING CALCULATOR ALLOWED

*****KNOW THE 12 PARENT FUNCTIONS PROPERTIES ... BE ABLE TO ANSWER QUESTION LIKE THE ONES ON WORKSHEET 3.

- 10. Are the given graphs functions? **Explain** why or why not.
 - Do they have an inverse that is also a function? **Explain** why or why not.



11. Given g(x) = 5x + 7 and $f(x) = 8 - 3x^2$, find g(f(-2)).

For Questions 12-19, suppose $f(x) = 2x^2 - 7$ and g(x) = 3x - 2. Find each of the following AND state the domain of the new function.

12. (f+g)(x) 13. (fg)(x) 14. $(\frac{f}{g})(x)$

15.
$$f^{-1}(x)$$
 16. $g(f(x))$ 17. $\left(\frac{g}{f}\right)(x)$

18. Reflection of f(x) over the *x*-axis

19. Reflection of g(x) over the *y*-axis

For Questions 20 and 21, prove algebraically whether the function is even, odd, or neither.

20.
$$f(x) = 7x^4 - x^2$$
 21. $f(x) = \frac{3}{4x}$

- 22. For each of the following,
 - i) Find the inverse of the function.
 - ii) VERIFY that the function is an inverse by showing f(g(x)) = x = g(f(x))

a)
$$f(x) = 3x - 2$$

b) $h(x) = \frac{2x+4}{5-x}$

For Questions 23-25, give the equation of the function whose graph is described.

- 23. The graph of $f(x) = x^2$ is reflected across the *x*-axis and vertically stretched by a factor of 7.
- 24. The graph of $f(x) = \sqrt{x}$ is shifted 10 units to the left and 7 units upwards.
- 25. The graph of f(x) = |x| is horizontally shrunk by a factor of 1/2.
- 26. Using your calculator, graph the function and
 - (a) State the intervals on which the function is increasing, decreasing, or constant.
 - (b) Find all relative maximum and minimum values.

$$f(x) = 2x^3 - 5$$

27. Joe Pearlman received a 3.5% pay raise. His salary after the raise was \$37,260. What was his salary before the raise?

28. Sue invested \$10,000, part at 3.6% annual interest and the balance at 7.8% annual interest. How much invested at each rate if she received a 1-year interest payment of \$667.02.

29. Thirty kilograms of nickel that was Thirty kilograms of nickel that was 38% pure were mixed with a nickel allow that was 28% pure. How may kilograms of each kind were used to produce an alloy that was 32% pure?

Can you...

- ... transform all the parent functions and graph them using correct points?
- ... graph a piecewise function?
- ... find the domain using the 3 domain issues discussed?
- ... find (and tell the difference) between a vertical asymptote and hole in a function when given the equation?
- ... describe the end behavior of a function using limit notation?
- ... find horizontal asymptotes of a rational function?
- ... use the function notation to add/subtract/multiply/divide functions?
- ... use composite function notation?
- ... prove a function is odd or even?
- ... find the inverse of an equation?
- ... prove an equation is an inverse, and know the difference between proving and finding inverses?
- ... correctly describe when a function is increasing or decreasing?
- ... correctly find and describe the extrema of a function?
- ... explain the difference between absolute and relative (local) extrema?
- ... use the terms bounded, bounded above, and bounded below correctly?
- ... describe the difference between removable and non-removable discontinuities?
- ... use properties of a graph to identify which parent function would model the graph best?
- ... do "mixture" problems?
- ... work any problem from the first 2 tests? (YEP!!! ... Anything from the first 2 tests could be on this test too! REMEMBER: all of this stuff ties together, so you never know when we will use it!) REVIEW ALL YOUR WORKSHEETS & NOTES!