AP Calculus AB Name___________________________

Mock AP Exam #3 Review

The Mock AP Exam

Thursday- Multiple Choice

There will be 5 Calculator Multiplice Choice Questions and 15 Non-Calculator Multiple Choice Questions. This portion of the Mock AP Exam is worth 10% of your Marking Period 3 grade.

Friday- Open Ended

There will be 1 Calculator Problem and 2 Non-Calculator Problems. This portion of the Mock AP Exam is also worth 10% of your Marking Period 3 grade.

Topics:

	Page in Packet	Review/New
1. Differentiability and Continuity	2-5	Review
2. Implicit Differentiation	6-10	Review
3. Derivative Tests a. First Derivative Test for Critical Points b. Second Derivative Test for Inflection Points c. Second Derivative Test for Critical Points	11-17	Review
4. Curve Sketching a. Find the graph of f from f’ b. Find the graph of f’ from f c. Determine critical points on the graph of f from the graph of f’ d. Determine maximums/minimums on the graph of f from graph of f’ e. Determine inflection points on the graph of f from the graph o f’	18-23	Review
5. Evaluating Derivatives and Integrals on a Calculator	24	New
6. Basic Antidifferentiation/Integration General and Particular Solutions	25-27	Review
7. Integration with u/du Substitution	28- 30	New
8. Riemann Sums a. Find area under curve using geometry b. Left, Right, Midpoint, and Trapezoidal approximations	31-33	New
9. Fundamental Theorem of Calculus- Evaluation a. Evaluate area under a curve using the FTC	34-36	New

1. Differentiability Multiple Choice Practice

Solutions found on Teacher Page under *AP Calculus AB Exploration Notes* Tab

"January 28- Investigating Implicit Differentiation- Day 2 Solutions"

Let $f(x)$ be the piecewise function defined below. At $x=2$, the function is ___________. (Choose the most correct answer to fill in the blank.)

$$f(x)=\begin{cases} x^2 & \text{for } x \le 2 \\ 8-2x & \text{for } x > 2 \end{cases}$$

a) Continuous but not differentiable **b)** Differentiable

c) Differentiable but not continuous **d)** Not continuous and not differentiable

$$f(x)=\begin{cases} \dfrac{x^2-4}{x-2} & \text{if } x \ne 2 \\ 1 & \text{if } x = 2 \end{cases}$$

6. Let f be the function defined above. Which of the following statements about f are true?

I. f has a limit at $x=2$.

II. f is continuous at $x=2$.

III. f is differentiable at $x=2$.

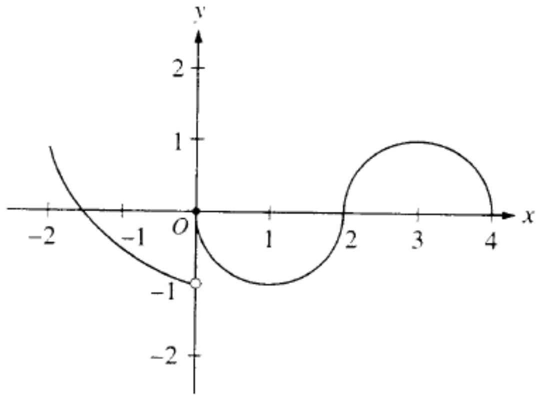
(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III



13. The graph of the function f shown in the figure above has a vertical tangent at the point $(2,0)$ and horizontal tangents at the points $(1,-1)$ and $(3,1)$. For what values of x, $-2<x<4$, is f not differentiable?

(A) 0 only (B) 0 and 2 only (C) 1 and 3 only (D) 0, 1, and 3 only (E) 0, 1, 2, and 3

$$f(x)=\begin{cases} x+2 & \text{if } x\le 3 \\ 4x-7 & \text{if } x>3 \end{cases}$$

20. Let *f* be the function given above. Which of the following statements about *f is false*?

I. $\lim_{x\to 3} f(x)$ exists.

II. f is continuous at $x=3$.

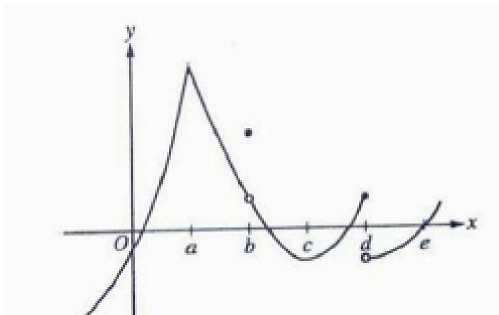
III. f is differentiable at $x=3$.

A) None
B) I only
C) II only
D) III only
E) I and II only

$$f(x)=\begin{cases} cx+d & \text{for } x\le 2 \\ x^2-cx & \text{for } x>2 \end{cases}$$

25. Let f be the function defined above, where c and d are constants. If f is differentiable at $x=2$, what is the value of $c+d$?

(A) -4 (B) -2 (C) 0 (D) 2 (E) 4



Graph of f

13. The graph of the function *f* is shown above. At which value(s) of *x* is f not differentiable?

A) a
B) a and b
C) a and d
D) b and d
E) $a, b,$ and d

16. Which of the following functions shows that the statement "If a function is continuous at $x=0$, then it is differentiable at $x=0$" is false?

(A) $f(x)=x^{-\frac{4}{3}}$ (B) $f(x)=x^{-\frac{1}{3}}$ (C) $f(x)=x^{\frac{1}{3}}$ (D) $f(x)=x^{\frac{4}{3}}$ (E) $f(x)=x^3$

27. At $x=3$, the function given by $f(x)=\begin{cases} x^2, & x<3 \\ 6x-9, & x\geq 3 \end{cases}$ is

(A) undefined.

(B) continuous but not differentiable.

(C) differentiable but not continuous.

(D) neither continuous nor differentiable.

(E) both continuous and differentiable.

41. If $\lim_{x\to 3} f(x)=7$, which of the following must be true?

I. f is continuous at $x=3$.
II. f is differentiable at $x=3$.
III. $f(3)=7$

(A) None (B) II only (C) III only

(D) I and III only (E) I, II, and III

81. Let f be the function given by $f(x)=|x|$. Which of the following statements about f are true?

I. f is continuous at $x=0$.
II. f is differentiable at $x=0$.
III. f has an absolute minimum at $x=0$.

(A) I only (B) II only (C) III only (D) I and III only (E) II and III only

2. Implicit Differentiation Practice

Not all of these problems require Implicit Differentiation to complete – be careful.
Solutions found on Teacher Page under *AP Calculus AB Exploration Notes* Tab
"January 29- dy-dx and the AP Exam Classwork Solutions"

1969 AB:

5. If $3x^2 + 2xy + y^2 = 2$, then the value of $\frac{dy}{dx}$ at $x = 1$ is

(A) -2 (B) 0 (C) 2 (D) 4 (E) not defined

24. If $\sin x = e^y$, $0 < x < \pi$, what is $\frac{dy}{dx}$ in terms of x?

(A) $-\tan x$ (B) $-\cot x$ (C) $\cot x$ (D) $\tan x$ (E) $\csc x$

39. If $y = \tan u$, $u = v - \frac{1}{v}$, and $v = \ln x$, what is the value of $\frac{dy}{dx}$ at $x = e$?

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) $\frac{2}{e}$ (E) $\sec^2 e$

1973

9. If $y = \cos^2 3x$, then $\frac{dy}{dx} =$

(A) $-6\sin 3x \cos 3x$ (B) $-2\cos 3x$ (C) $2\cos 3x$

(D) $6\cos 3x$ (E) $2\sin 3x \cos 3x$

40. If $\tan(xy) = x$, then $\frac{dy}{dx} =$

(A) $\frac{1 - y\tan(xy)\sec(xy)}{x\tan(xy)\sec(xy)}$ (B) $\frac{\sec^2(xy) - y}{x}$ (C) $\cos^2(xy)$

(D) $\frac{\cos^2(xy)}{x}$ (E) $\frac{\cos^2(xy) - y}{x}$

1985

3. If $y = \frac{3}{4+x^2}$, then $\frac{dy}{dx} =$

(A) $\frac{-6x}{\left(4+x^2\right)^2}$ (B) $\frac{3x}{\left(4+x^2\right)^2}$ (C) $\frac{6x}{\left(4+x^2\right)^2}$ (D) $\frac{-3}{\left(4+x^2\right)^2}$ (E) $\frac{3}{2x}$

13. If $x^2 + xy + y^3 = 0$, then, in terms of x and y, $\frac{dy}{dx} =$

(A) $-\frac{2x+y}{x+3y^2}$ (B) $-\frac{x+3y^2}{2x+y}$ (C) $\frac{-2x}{1+3y^2}$ (D) $\frac{-2x}{x+3y^2}$ (E) $-\frac{2x+y}{x+3y^2-1}$

1988

1. If $y = x^2 e^x$, then $\frac{dy}{dx} =$

(A) $2xe^x$ (B) $x\left(x+2e^x\right)$ (C) $xe^x\left(x+2\right)$

(D) $2x+e^x$ (E) $2x+e$

6. If $y = \frac{\ln x}{x}$, then $\frac{dy}{dx} =$

(A) $\frac{1}{x}$ (B) $\frac{1}{x^2}$ (C) $\frac{\ln x - 1}{x^2}$ (D) $\frac{1-\ln x}{x^2}$ (E) $\frac{1+\ln x}{x^2}$

9. If $x + 2xy - y^2 = 2$, then at the point $(1,1)$, $\frac{dy}{dx}$ is

(A) $\frac{3}{2}$ (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{3}{2}$ (E) nonexistent

18. If $y = 2\cos\left(\frac{x}{2}\right)$, then $\frac{d^2y}{dx^2} =$

(A) $-8\cos\left(\frac{x}{2}\right)$ (B) $-2\cos\left(\frac{x}{2}\right)$ (C) $-\sin\left(\frac{x}{2}\right)$ (D) $-\cos\left(\frac{x}{2}\right)$ (E) $-\frac{1}{2}\cos\left(\frac{x}{2}\right)$

1988 BC

6. If $y^2 - 2xy = 16$, then $\frac{dy}{dx} =$

(A) $\frac{x}{y-x}$ (B) $\frac{y}{x-y}$ (C) $\frac{y}{y-x}$ (D) $\frac{y}{2y-x}$ (E) $\frac{2y}{x-y}$

1993

4. If $x^3+3xy+2y^3=17$, then in terms of x and y, $\frac{dy}{dx}=$

(A) $-\frac{x^2+y}{x+2y^2}$

(B) $-\frac{x^2+y}{x+y^2}$

(C) $-\frac{x^2+y}{x+2y}$

(D) $-\frac{x^2+y}{2y^2}$

(E) $\frac{-x^2}{1+2y^2}$

8. If $y=\tan x-\cot x$, then $\frac{dy}{dx}=$

(A) $\sec x\csc x$ (B) $\sec x-\csc x$ (C) $\sec x+\csc x$ (D) $\sec^2 x-\csc^2 x$ (E) $\sec^2 x+\csc^2 x$

1998

6. If $x^2+xy=10$, then when $x=2$, $\frac{dy}{dx}=$

(A) $-\frac{7}{2}$ (B) -2 (C) $\frac{2}{7}$ (D) $\frac{3}{2}$ (E) $\frac{7}{2}$

3. Derivative Test AP Problems

Solutions found on Teacher Page under *AP Calculus AB Exploration Notes* Tab
"February 3- Derivative Test AP Problems- Solutions"

1973

80. The derivative of the function f is given by $f'(x) = x^2 \cos\left(x^2\right)$. How many points of inflection does the graph of f have on the open interval $(-2, 2)$?

(A) One (B) Two (C) Three (D) Four (E) Five

1977

22. Given the function defined by $f(x) = 3x^5 - 20x^3$, find all values of x for which the graph of f is concave up.

(A) $x > 0$
(B) $-\sqrt{2} < x < 0$ or $x > \sqrt{2}$
(C) $-2 < x < 0$ or $x > 2$
(D) $x > \sqrt{2}$
(E) $-2 < x < 2$

1985

39. If $f(x) = \dfrac{\ln x}{x}$, for all $x > 0$, which of the following is true?

(A) f is increasing for all x greater than 0.
(B) f is increasing for all x greater than 1.
(C) f is decreasing for all x between 0 and 1.
(D) f is decreasing for all x between 1 and e.
(E) f is decreasing for all x greater than e.

1988

4. The graph of $y = \dfrac{-5}{x-2}$ is concave downward for all values of x such that

(A) $x < 0$ (B) $x < 2$ (C) $x < 5$ (D) $x > 0$ (E) $x > 2$

1993

15. For what value of x does the function $f(x) = (x-2)(x-3)^2$ have a relative maximum?

(A) -3 (B) $-\dfrac{7}{3}$ (C) $-\dfrac{5}{2}$ (D) $\dfrac{7}{3}$ (E) $\dfrac{5}{2}$

19. Let f be the function defined by $f(x) = \begin{cases} x^3 & \text{for } x \le 0, \\ x & \text{for } x > 0. \end{cases}$ Which of the following statements about f is true?

(A) f is an odd function.

(B) f is discontinuous at $x = 0$.

(C) f has a relative maximum.

(D) $f'(0) = 0$

(E) $f'(x) > 0$ for $x \ne 0$

23. How many critical points does the function $f(x)=(x+2)^5(x-3)^4$ have?

(A) One (B) Two (C) Three (D) Five (E) Nine

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27. The function f given by $f(x)=x^3+12x-24$ is

(A) increasing for $x<-2$, decreasing for $-2<x<2$, increasing for $x>2$
(B) decreasing for $x<0$, increasing for $x>0$
(C) increasing for all x
(D) decreasing for all x
(E) decreasing for $x<-2$, increasing for $-2<x<2$, decreasing for $x>2$

1997

5. The graph of $y=3x^4-16x^3+24x^2+48$ is concave down for

(A) $x<0$

(B) $x>0$

(C) $x<-2$ or $x>-\frac{2}{3}$

(D) $x<\frac{2}{3}$ or $x>2$

(E) $\frac{2}{3}<x<2$

13. Let f be a function defined for all real numbers x. If $f'(x) = \dfrac{|4-x^2|}{x-2}$, then f is decreasing on the interval

(A) $(-\infty, 2)$ (B) $(-\infty, \infty)$ (C) $(-2, 4)$ (D) $(-2, \infty)$ (E) $(2, \infty)$

22. What are all values of x for which the function f defined by $f(x) = (x^2 - 3)e^{-x}$ is increasing?

(A) There are no such values of x.

(B) $x < -1$ and $x > 3$

(C) $-3 < x < 1$

(D) $-1 < x < 3$

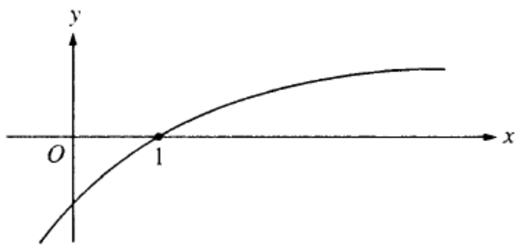
(E) All values of x

Calculator

77. The graph of the function $y = x^3 + 6x^2 + 7x - 2\cos x$ changes concavity at $x =$

(A) -1.58 (B) -1.63 (C) -1.67 (D) -1.89 (E) -2.33

1998



17. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f(1) < f'(1) < f''(1)$

(B) $f(1) < f''(1) < f'(1)$

(C) $f'(1) < f(1) < f''(1)$

(D) $f''(1) < f(1) < f'(1)$

(E) $f''(1) < f'(1) < f(1)$

22. The function f is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is f increasing?

(A) $\left(-\frac{1}{\sqrt{2}}, \infty\right)$

(B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(C) $(0, \infty)$

(D) $(-\infty, 0)$

(E) $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$

89. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = \left(x^2 - 4\right)g(x)$, which of the following is true?

(A) f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.
(B) f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.
(C) f has relative minima at $x = -2$ and at $x = 2$.
(D) f has relative maxima at $x = -2$ and at $x = 2$.
(E) It cannot be determined if f has any relative extrema.

2003

15. Let f be the function with derivative given by $f'(x) = x^2 - \frac{2}{x}$. On which of the following intervals is f decreasing?

A) $(-\infty, -1]$ only

B) $(-\infty, 0)$

C) $[-1, 0)$ only

D) $\left(0, \sqrt[3]{2}\right]$

E) $\left[\sqrt[3]{2}, \infty\right)$

x	-4	-3	-2	-1	0	1	2	3	4
$g'(x)$	2	3	0	-3	-2	-1	0	3	2

18. The derivative g' of a function g is continuous and has exactly two zeros. Selected values of g' are given in the table above. If the domain of g is the set of all real numbers, then g is decreasing on which of the following intervals?

A) $-2 \leq x \leq 2$ only

B) $-1 \leq x \leq 1$ only

C) $x \geq -2$

D) $x \geq 2$ only

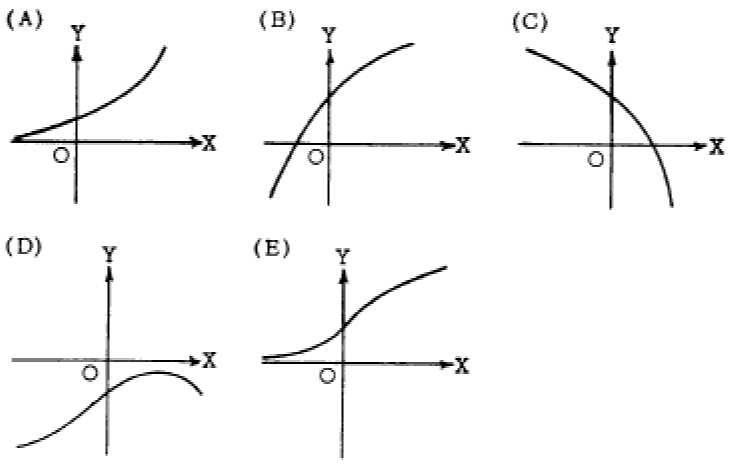
E) $x \leq -2$ or $x \geq 2$

4. The Relationship between the Graphs of $f, f', and\ f''$ AP Problems

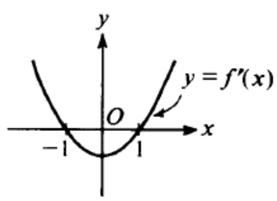
Solutions found on Teacher Page under *AP Calculus AB Exploration Notes* Tab
"February 18- The Relationship between the Graphs of f- f prime and f double prime AP Problems- Solutions"

1969 AB

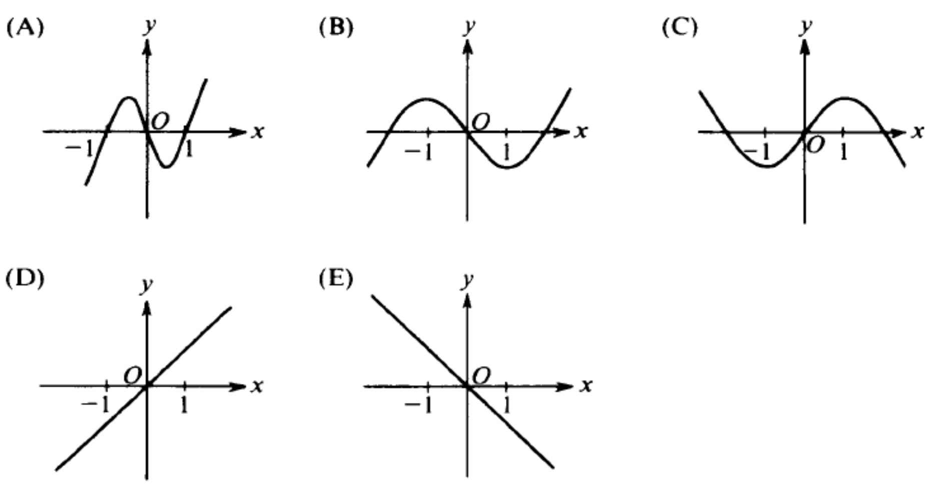
16. If y is a function x such that $y' > 0$ for all x and $y'' < 0$ for all x, which of the following could be part of the graph of $y = f(x)$?



1985 AB

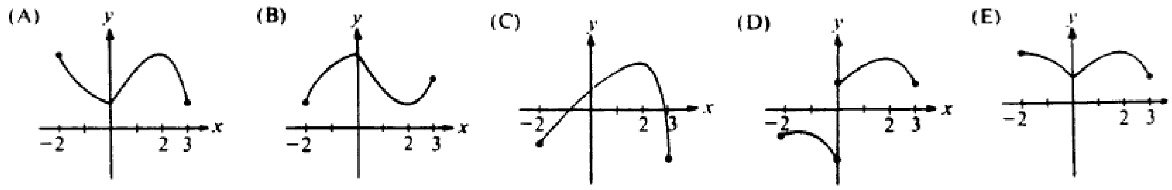


33. The graph of the <u>derivative</u> of f is shown in the figure above. Which of the following could be the graph of f?

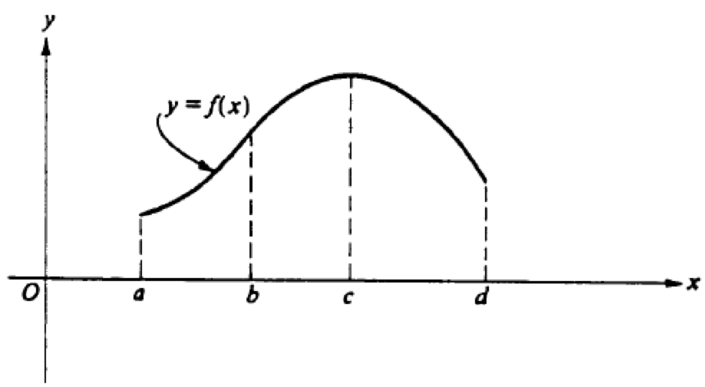


1985 BC

43. Let f be a function that is continuous on the closed interval $[-2,3]$ such that $f'(0)$ does not exist, $f'(2)=0$, and $f''(x)<0$ for all x except $x=0$. Which of the following could be the graph of f?



1988 AB



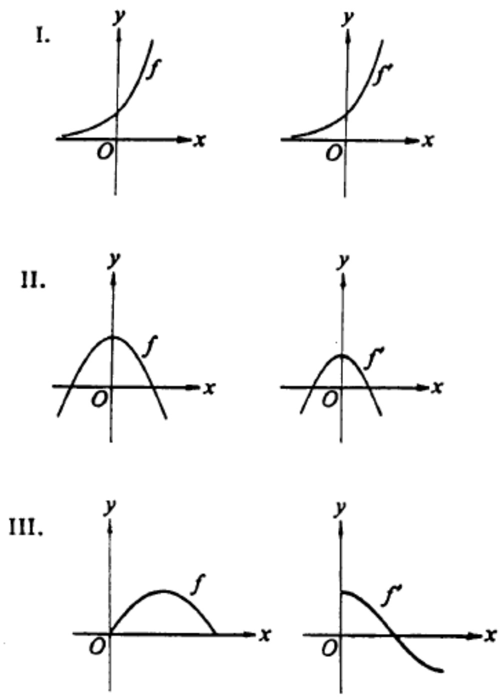
8. The graph of $y=f(x)$ is shown in the figure above. On which of the following intervals are $\frac{dy}{dx}>0$ and $\frac{d^2y}{dx^2}<0$?

I. $a<x<b$
II. $b<x<c$
III. $c<x<d$

(A) I only (B) II only (C) III only (D) I and II (E) II and III

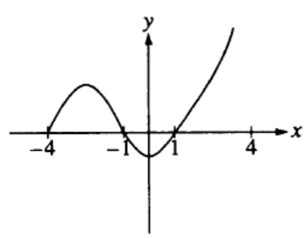
1988 BC

9. Which of the following pairs of graphs could represent the graph of a function and the graph of its derivative?



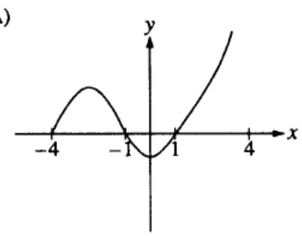
(A) I only (B) II only (C) III only (D) I and III (E) II and III

1993 AB

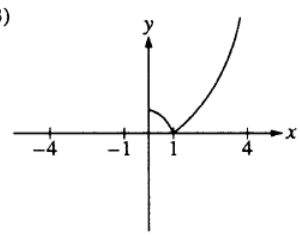


40. The graph of $y = f(x)$ is shown in the figure above. Which of the following could be the graph of $y = f(|x|)$?

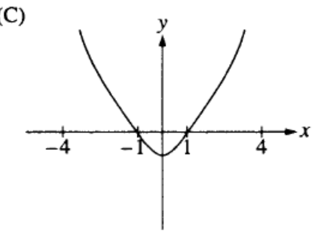
(A)



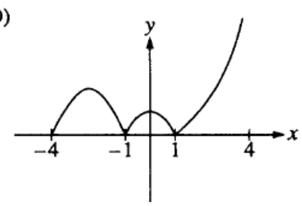
(B)



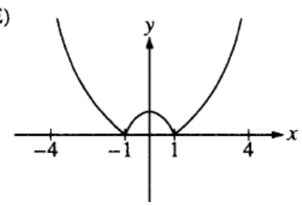
(C)

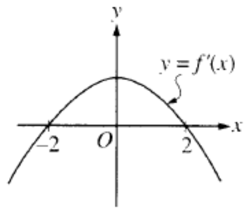


(D)



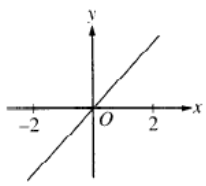
(E)



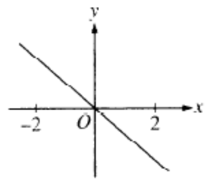


11. The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

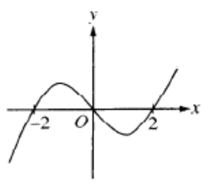
(A)



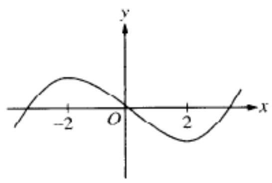
(B)



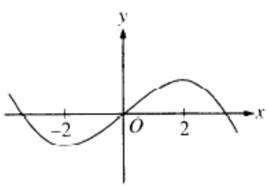
(C)



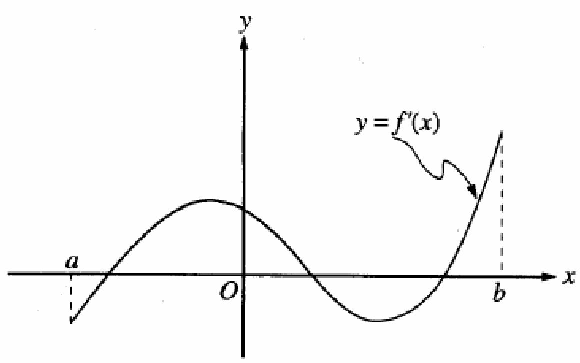
(D)



(E)

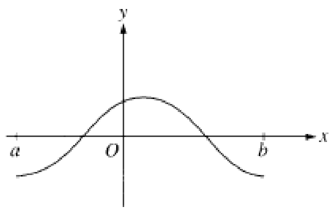


1997 BC



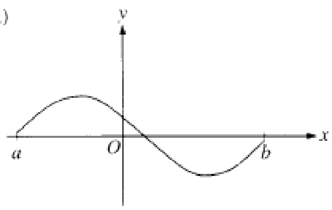
12. The graph of f', the derivative of f, is shown in the figure above. Which of the following describes all relative extrema of f on the open interval (a, b)?

(A) One relative maximum and two relative minima
(B) Two relative maxima and one relative minimum
(C) Three relative maxima and one relative minimum
(D) One relative maximum and three relative minima
(E) Three relative maxima and two relative minima

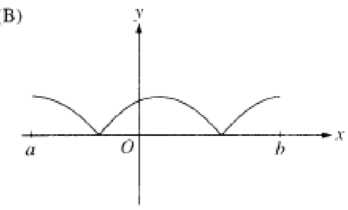


23. The graph of f is shown in the figure above. Which of the following could be the graph of the derivative of f?

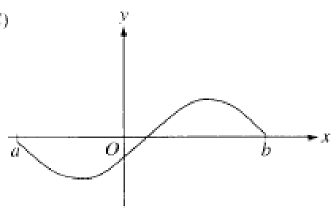
(A)



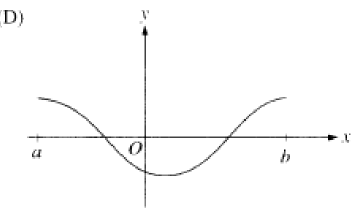
(B)



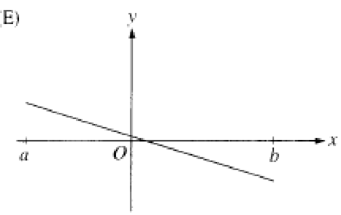
(C)

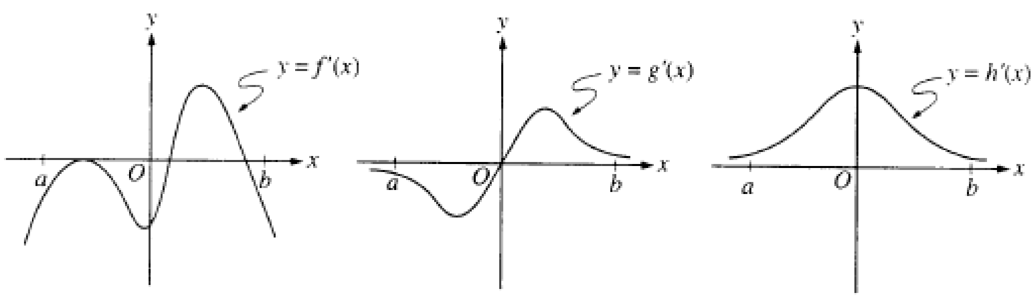


(D)



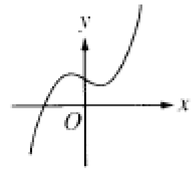
(E)





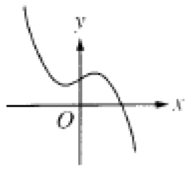
79. The graphs of the derivatives of the functions f, g, and h are shown above. Which of the functions f, g, or h have a relative maximum on the open interval $a < x < b$?

(A) f only
(B) g only
(C) h only
(D) f and g only
(E) f, g, and h

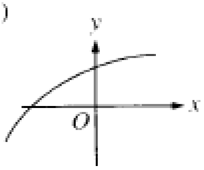


6. The graph of $y = h(x)$ is shown above. Which of the following could be the graph of $y = h'(x)$?

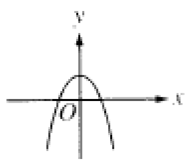
(A)



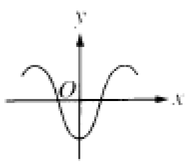
(B)



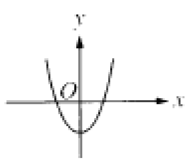
(C)



(D)



(E)



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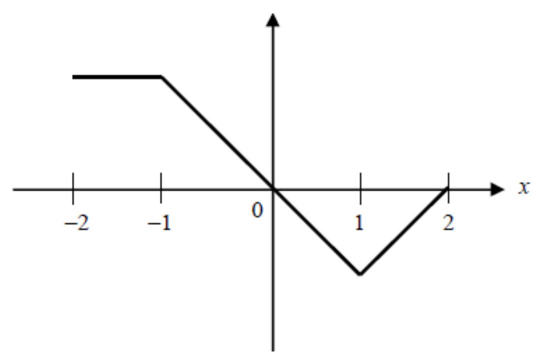

Graph of f'

7. The graph of f', the derivative of the function f, is shown above. Which of the following statements is true about f?

A) f is decreasing for $-1 \le x \le 1$.

B) f is increasing for $-2 \le x \le 0$.

C) f is increasing for $1 \le x \le 2$.

D) f has a local minimum at x = 0.

E) f is not differentiable at x = -1 and x = 1.

5. Evaluating Derivatives and Integrals on a Calculator

This is a reminder of how to use your calculator to evaluate derivatives and find definite integrals. Use these methods to check your work. You may need to use these methods on the calculator portion of the exam.

Derivatives Using the Calculator

Ex. 2 Evaluate the derivative at the given value of x.

$$f(x)=\frac{x+2}{x-2}; \quad x=3$$

- $\boxed{Y=}$ $\boxed{(x+2)\div(x-2)}$
- 2^{nd} $\overset{\text{QUIT}}{\boxed{\text{MODE}}}$
- $\boxed{\text{MATH}}$ 8:nDeriv(
- $\boxed{\text{VARS}}$ → Y-VARS "1: Function" "1: Y_1" $\boxed{,}$ $\boxed{X,T,\theta,n}$ $\boxed{,}$ 3 $\boxed{)}$

7	Evaluate $f'(2)$ for $f(x)=x\tan x$.
8	Find the equation of the tangent line for the function $f(x)=x^3-4x^2+2x-1$ when $x=1$.
9	Find the instantaneous rate of change of $f(x)=\frac{x}{\sin x}$ when $x=\frac{\pi}{3}$.

Integrals Using a Calculator

Ex. Evaluate the integral using a calculator. Your answer should be correct to **three** numbers after the decimal.

$$\int_0^4 \sqrt{x}dx$$

- $\boxed{\text{Y=}}$ $\sqrt{x}$
- $\boxed{2^{nd}}$ $\overset{\text{QUIT}}{\boxed{\text{MODE}}}$
- $\boxed{\text{MATH}}$ 9: fnInt(
- $\boxed{\text{VARS}}$ → Y-VARS "1: Function" "1: Y_1" $\boxed{,}$ $\boxed{X,T,\theta,n}$ $\boxed{,}$ 0 $\boxed{,}$ 4 $\boxed{)}$

1	Evaluate $\int_1^e x^2 \ln x dx$ using your calculator. Your answer should be correct to three decimal places.
2	Evaluate $\int_0^2 (2x-\ln x)dx$ using your calculator. Your answer should be correct to three decimal places.

6. Basic Antidifferentiation/Integration General and Particular Solutions

Solutions found on Teacher Page under *AP Calculus AB Exploration Notes* Tab
"March 4- Antiderivatives and Initial Value Problems MC Practice- Solutions"

1) $\int \pi^3 \, dx =$

A) $3\pi^2 x + c$
B) 0
C) $\pi^3 x + c$
D) $3\pi^2 + c$
E) $\frac{\pi^4}{4} + c$

4) $\int (x^2 - 2)^2 \, dx =$

A) $\left(\frac{x^2 - 2}{3}\right)^3 + c$
B) $\frac{(x^2 - 2)^3}{6x} + c$
C) $\frac{2x}{3}(x^2 - 2)^3 + c$
D) $\left(\frac{x^3}{3} - 2x\right)^2 + c$
E) $\frac{x^5}{5} - \frac{4x^3}{3} + 4x + c$

2) $\int (x^4 - x^3 + x^2) \, dx =$

A) $\frac{x^5}{4} - \frac{x^4}{3} + \frac{x^3}{2} + c$
B) $5x^5 - 4x^4 + 3x^3 + c$
C) $\frac{x^5}{5} - 3x^2 + \frac{x^3}{3} + c$
D) $4x^3 - 3x^2 + 2x + c$
E) $\frac{x^5}{5} - \frac{x^4}{4} + \frac{x^3}{3} + c$

5) $\int x^3 (x^3 - 2) \, dx =$

A) $\frac{x^7}{7} - \frac{x^4}{2} + c$
B) $\frac{x^4}{4}\left(\frac{x^4}{4} - 2x\right) + c$
C) $\frac{3(x^3 - 3)^2}{2} + c$
D) $3x^2\left(\frac{x^4}{4} - 2x\right) + c$
E) $6x^5 - 6x^2 + c$

3) $\int (x^2 + 2)(1 - x) \, dx =$

A) $\frac{x^3}{3} - 2x^2 + c$
B) $-\frac{x^4}{4} + \frac{x^3}{3} - x^2 + c$
C) $-3x^2 + 2x - 2 + c$
D) $-\frac{x^4}{4} + \frac{x^3}{3} - x^2 + 2x + c$
E) $\left(\frac{x^3}{3} - 2x\right)(x - x^2) + c$

6) $\int \frac{3x^5 + 2x^3 - x^2}{x^2} \, dx =$

A) $\frac{x^6 + x^4 - x^3}{6x^3} + c$
B) $\frac{15x^4 + 6x^2 - 2x}{2x} + c$
C) $18x^6 + 8x^2 - 2x + c$
D) $3x^4 + 2x^2 - x + c$
E) $\frac{3}{4}x^4 + x^2 - x + c$

7) $\int \frac{1}{\sqrt{x}}\,dx =$

A) $\frac{1}{2}x\sqrt{x} + c$

B) $\frac{2}{3}x\sqrt{x} + c$

C) $-\frac{1}{2}x\sqrt{x} + c$

D) $2\sqrt{x} + c$

E) $\frac{1}{2}\sqrt{x} + c$

8) $\int \frac{1}{\sqrt[3]{x^2}}\,dx =$

A) $\frac{3}{2}x^{\frac{2}{3}} + c$

B) $-\frac{1}{3}x^{-\frac{1}{3}} + c$

C) $3x^{\frac{1}{3}} + c$

D) $\frac{1}{3}x^{\frac{1}{3}} + c$

E) $-3x^{-\frac{1}{3}} + c$

9) $\int \left(\frac{3}{u^{\frac{3}{4}}} - 4\sqrt[3]{u} + 1 \right) du =$

A) $\frac{21}{4u^{\frac{7}{4}}} - \frac{1}{3}u^{\frac{4}{3}} + u + c$

B) $\frac{3}{4}u^{\frac{1}{4}} - \frac{16}{3}u^{\frac{4}{3}} + u + c$

C) $12u^{\frac{3}{4}} - 3u^{\frac{5}{3}} + u + c$

D) $\frac{4}{3}u^{\frac{1}{4}} - \frac{3}{16}u^{\frac{4}{3}} + u + c$

E) $12u^{\frac{1}{4}} - 3u^{\frac{4}{3}} + u + c$

10) $\int \left(\sqrt[4]{x^3} - \frac{2}{\sqrt[3]{x^2}} \right) dx =$

A) $\frac{3}{4\sqrt[4]{x}} - \frac{4}{3\sqrt[3]{x}} + c$

B) $\frac{4}{7}\sqrt[4]{x^3} - \frac{2}{3}\sqrt[3]{x} + c$

C) $\frac{4}{7}x\sqrt[4]{x^3} - 6\sqrt[3]{x} + c$

D) $\frac{4}{7}x\sqrt[4]{x^3} - \frac{6}{5}x\sqrt[3]{x^2} + c$

E) $\frac{7}{4}x\sqrt[4]{x^3} - \frac{5}{6}x\sqrt[3]{x^2} + c$

11) If $g'(x) = 4x^3 + 3x^2 + 6x$ and $g(1) = -3$, then $g(x) =$

A) $x^4 + x^3 + 6x^2 - 11$

B) $\frac{x^4}{4} + \frac{x^3}{3} + 3x^2 - \frac{79}{12}$

C) $12x^2 + 6x + 6$

D) $x^4 + x^3 + 3x^2$

E) $x^4 + x^3 + 3x^2 - 8$

12) Which of the following defines a function f such that $f'(x) = \sqrt{x}$ and the graph of function f pass through the point (9,0)?

A) $f(x) = \frac{2}{3}x\sqrt{x} - 18$

B) $f(x) = x\sqrt{x} - 3x$

C) $f(x) = \frac{x\sqrt{x}}{3} + 9$

D) $f(x) = \frac{1}{2}\sqrt{x} - 3$

E) $f(x) = \frac{3}{2}x\sqrt{x} - 18$

13) The slope of a curve at each point (x,y) is given by $4x - 1$. Which of the following is an equation for this curve if it passes through the point (-2,3)?

A) $y = 2x^2 - x - 7$

B) $y = 4x^2 - x - 15$

C) $y = 2x^2 - x + 7$

D) $y = x^2 - 4x - 9$

E) $y = 2x^2 - x$

14) If function f has a derivative defined by $f'(x) = \frac{x+1}{\sqrt{x}}$ and $f(1) = 0$, then $f(4) =$

A) $\frac{20}{3}$

B) $-\frac{4}{3}$

C) $\frac{4}{3}$

D) $-\frac{8}{3}$

E) $\frac{3}{4}$

15) The slope of the line tangent to the graph of a function f at any point (x,y) is given by $x^3 - x$. If the graph of function f passes through the point (2,1), find f(0).

A) 1

B) 2

C) 3

D) -1

E) 0

16) A function f has a derivative $f'(x) = 3 - 2x$. An equation of the line tangent to the graph of function f at $x = 2$ is $y - 7 = -(x - 2)$. What is an equation of function f?

A) $f(x) = -x^2 + 3x$

B) $f(x) = -3x^2 + x - 3$

C) $f(x) = -x^2 + 3x - 3$

D) $f(x) = 3x^2 + 3x - 1$

E) $f(x) = x^2 - 3x + 3$

17) If $h''(x) = x - 2$, $h'(4) = 0$, and $h(0) = 4$, then $h(x) =$

A) $\frac{x^3}{6} - x^2 + 4$

B) $2x^3 - x^2 + 4$

C) $\frac{x^3}{3} - \frac{x^2}{2} + 4$

D) $\frac{x^3}{6} - x^2$

E) $\frac{1}{2}x^2 + 2x + 4$

18) At each point (x,y) on a curve, $\frac{d^2y}{dx^2} = 6x$. Additionally, the line $y = 6x + 4$ is tangent to the curve at $x = -2$. Which of the following is an equation of the curve that satisfies these conditions?

A) $y = 6x^2 - 32$

B) $y = x^3 - 6x - 12$

C) $y = 2x^3 - 3x$

D) $y = x^3 - 6x + 12$

E) $y = 2x^3 + 3x - 12$

4. $\int \sin(3x+4)\,dx =$

(A) $-\frac{1}{3}\cos(3x+4)+C$
(B) $-\cos(3x+4)+C$
(C) $-3\cos(3x+4)+C$
(D) $\cos(3x+4)+C$
(E) $\frac{1}{3}\cos(3x+4)+C$

12. $\int (3x+5)^2\,dx =$

(A) $\frac{1}{3}(3x+5)^3+C$
(B) $2(3x+5)+C$
(C) $6(3x+5)+C$
(D) $\frac{1}{9}(3x+5)^3+C$
(E) $\frac{1}{9}(3x+5)+C$

97 6. $\frac{1}{2}\int e^{\frac{t}{2}}\,dt =$

(A) $e^{-t}+C$ (B) $e^{-\frac{t}{2}}+C$ (C) $e^{\frac{t}{2}}+C$ (D) $2e^{\frac{t}{2}}+C$ (E) $e^{t}+C$

69 38. $\int \frac{x^2}{e^{x^3}}\,dx =$

(A) $-\frac{1}{3}\ln e^{x^3}+C$ (B) $-\frac{e^{x^3}}{3}+C$ (C) $-\frac{1}{3e^{x^3}}+C$
(D) $\frac{1}{3}\ln e^{x^3}+C$ (E) $\frac{x^3}{3e^{x^3}}+C$

93 BC 20. $\int x\sqrt{4-x^2}\,dx =$

(A) $\dfrac{\left(4-x^2\right)^{3/2}}{3}+C$ (B) $-\left(4-x^2\right)^{3/2}+C$ (C) $\dfrac{x^2\left(4-x^2\right)^{3/2}}{3}+C$

(D) $-\dfrac{x^2\left(4-x^2\right)^{3/2}}{3}+C$ (E) $-\dfrac{\left(4-x^2\right)^{3/2}}{3}+C$

88 7. $\int \dfrac{x\,dx}{\sqrt{3x^2+5}} =$

(A) $\frac{1}{9}\left(3x^2+5\right)^{\frac{3}{2}}+C$ (B) $\frac{1}{4}\left(3x^2+5\right)^{\frac{3}{2}}+C$ (C) $\frac{1}{12}\left(3x^2+5\right)^{\frac{1}{2}}+C$

(D) $\frac{1}{3}\left(3x^2+5\right)^{\frac{1}{2}}+C$ (E) $\frac{3}{2}\left(3x^2+5\right)^{\frac{1}{2}}+C$

93 14. $\int \dfrac{3x^2}{\sqrt{x^3+1}}\,dx =$

(A) $2\sqrt{x^3+1}+C$

(B) $\frac{3}{2}\sqrt{x^3+1}+C$

(C) $\sqrt{x^3+1}+C$

(D) $\ln\sqrt{x^3+1}+C$

(E) $\ln(x^3+1)+C$

69 43. $\int \sin(2x+3)\,dx =$

(A) $\frac{1}{2}\cos(2x+3)+C$ (B) $\cos(2x+3)+C$ (C) $-\cos(2x+3)+C$

(D) $-\frac{1}{2}\cos(2x+3)+C$ (E) $-\frac{1}{5}\cos(2x+3)+C$

I IC

Does not require u/du:

93 38. If the second derivative of f is given by $f''(x)=2x-\cos x$, which of the following could be $f(x)$?

(A) $\frac{x^3}{3}+\cos x-x+1$

(B) $\frac{x^3}{3}-\cos x-x+1$

(C) $x^3+\cos x-x+1$

(D) $x^2-\sin x+1$

(E) $x^2+\sin x+1$

Recall $tanx = \frac{sinx}{cosx}$

85 30. $\int \tan(2x)\,dx =$

(A) $-2\ln|\cos(2x)|+C$ (B) $-\frac{1}{2}\ln|\cos(2x)|+C$ (C) $\frac{1}{2}\ln|\cos(2x)|+C$

(D) $2\ln|\cos(2x)|+C$ (E) $\frac{1}{2}\sec(2x)\tan(2x)+C$

No u/du required:

88 5. $\int \sec^2 x\,dx =$

(A) $\tan x+C$ (B) $\csc^2 x+C$ (C) $\cos^2 x+C$

(D) $\frac{\sec^3 x}{3}+C$ (E) $2\sec^2 x\tan x+C$

1. Using a left Riemann sum, what is the area under the curve $y = 2x - x^2$ from x = 1 to x =2, with 4 subintervals.

(A) .53125 (B) .65625 (C) .66667 (D) .671875 (E) .78125

2. Using a left Riemann sum, what is the area under the curve $y = \sqrt{x}$ from x = 1 to x = 3, with 4 subintervals.

(A) 2.976 (B) 2.800 (C) 2.797 (D) 2.793 (E) 2.610

3. Using a left Riemann sum, what is the area under the curve $y = x^2 + x$ from x = 0 to x = 3, with 6 subintervals.

(A) 10.625 (B) 13.4375 (C) 13.5 (D) 13.625 (E) 16.625

5. Using a right Riemann sum, what is the area under the curve $y = \sqrt{x}$ from x = 1 to x = 3, with 4 subintervals.

(A) 2.610 (B) 2.793 (C) 2.797 (D) 2.800 (E) 2.976

6. Using a right Riemann sum, what is the area under the curve $y = 2x - x^2$ from x = 1 to x = 2, with 4 subintervals.

(A) .53125 (B) .65625 (C) .66667 (D) .67187 (E) .78125

7. Using a right Riemann sum, what is the area under the curve $y = x^2 + x$ from x = 0 to x = 3, with 6 subintervals.

(A) 10.625 (B) 13.4375 (C) 13.5 (D) 13.625 (E) 16.625

8. Using a right Riemann sum, what is the area under the curve $y = \sin x$ from x = 0 to $x = \pi$, with 4 subintervals.

(A) 2.104 (B) 2.000 (C) 2.052 (D) 1.948 (E) 1.896

9. For the function whose values are given in the table to the right, approximate $\int_0^4 f(x)\,dx$ using a Riemann Sum with a midpoint rule of two intervals.

x	0	1	2	3	4
f(x)	0	1.2	4.3	6.5	1

(A) 15.4 (B) 12.5 (C) 12 (D) 16.7 (E) 13

10. Using the Midpoint Formula, what is the area under the curve $y = \sqrt{x}$ from x = 1 to x = 3, using 4 subintervals?

(A) 2.610 (B) 2.793 (C) 2.797 (D) 2.800 (E) 2.976

15. Using the midpoint formula, what is the area under the curve $y = 2x - x^2$ from x = 1 to x = 2, using 4 subintervals?

(A) .53125 (B) .65625 (C) .66667 (D) .67187 (E) .78125

16. Using the trapezoidal rule, what is the area under the curve $y = \sqrt{x}$ from x = 1 to x = 3, using 4 subintervals?

(A) 2.610 (B) 2.793 (C) 2.797 (D) 2.800 (E) 2.976

20. Using the trapezoidal rule, what is the area under the curve y = sinx from x = 0 to $x = \pi$, using 4 subintervals?

(A) 1.896 (B) 1.948 (C) 2.000 (D) 2.052 (E) 2.104

21. Using the trapezoidal rule, what is the area under the curve $y = 2x - x^2$ from x = 1 to x = 2, using 4 subintervals?

(A) 0.53125 (B) 0.65625 (C) 0.66667 (D) 0.67187 (E) 0.78125

24. Which of the following are true?

I. If f(x) is increasing on [a, b], then the left Riemann sum for $\int_a^b f(x)\,dx$ is an underestimate.

II. If f(x) is increasing on [a, b] and g(x) is decreasing on [a, b], then the left Riemann sum for $\int_a^b f(x)\,dx > \int_a^b g(x)\,dx$.

III. If f(x) is concave down on [a, b], then the midpoint Riemann sum for $\int_a^b f(x)\,dx$ is an overestimate.

(A) I only (B) II only (C) I and III (D) II and III (E) I, II and III

13. For the function whose values are given in the table below, $\int_0^6 f(x)\,dx$ is approximated by a Riemann Sum using the value at the midpoint of each of three intervals of width 2.

x	0	1	2	3	4	5	6
$f(x)$	0	0.25	0.48	0.68	0.84	0.95	1

The approximation is

(A) 2.64
(B) 3.64
(C) 3.72
(D) 3.76
(E) 4.64

14. The table below gives selected values of a decreasing, differentiable function $f(x)$. Approximate the value of $\int_0^8 f(x)\,dx$ using a trapezoidal sum with four subintervals indicated by the data in the table.

x	0	1	5	6	8
$f(x)$	20	16	8	6	2

(A) 41
(B) 46
(C) 50
(D) 81
(E) 82

1. Which of the following properties of the definite integral are true?

I. $\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx,\ k$ is a constant

II. $\int_a^b x\,f(x)\,dx = x\int_a^b f(x)\,dx$

III. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

(A) I only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

2. $\int_0^2 (2x^3+3)\,dx =$

(A) 8
(B) 11
(C) 14
(D) 20
(E) 24

3. If $\int_2^8 f(x)\,dx = -10$ and $\int_2^4 f(x)\,dx = 6$, then $\int_4^8 f(x)\,dx =$

(A) –16
(B) –6
(C) –4
(D) 4
(E) 16

11. The area of the region between the graph of $y = 3x^2 + 2x$ and the x-axis from $x = 1$ to $x = 3$ is

(A) 36
(B) 34
(C) 31
(D) 26
(E) 12

1. $\int_{-1}^{1} (x^2 - x - 1)\, dx =$

(A) $\frac{2}{3}$ (B) 0 (C) $-\frac{4}{3}$ (D) -2 (E) -1

2. $\int_{1}^{2} \frac{3x-1}{3x}\, dx =$

(A) $\frac{3}{4}$ (B) $1 - \frac{1}{3}\ln 2$ (C) $1 - \ln 2$ (D) $-\frac{1}{3}\ln 2$ (E) 1

3. $\int_{0}^{3} \frac{dt}{\sqrt{4-t}} =$

(A) 1 (B) -2 (C) 4 (D) -1 (E) 2

5. $\int_{2}^{3} \frac{dy}{2y-3} =$

(A) $\ln 3$ (B) $\frac{1}{2}\ln\frac{3}{2}$ (C) $\frac{16}{9}$ (D) $\ln\sqrt{3}$ (E) $\sqrt{3} - 1$

6. $\int_{0}^{\sqrt{3}} \frac{x}{\sqrt{4-x^2}}\, dx =$

(A) 1 (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) -1 (E) 2

7. $\int_{0}^{1} (2t-1)^3\, dt =$

(A) $\frac{1}{4}$ (B) 6 (C) $\frac{1}{2}$ (D) 0 (E) 4

8. $\int_{4}^{9} \frac{2+x}{2\sqrt{x}}\, dx =$

(A) $\frac{25}{3}$ (B) $\frac{41}{3}$ (C) $\frac{100}{3}$ (D) $\frac{5}{3}$ (E) $\frac{1}{3}$

10. $\int_0^1 e^{-x}\,dx =$

(A) $\frac{1}{e} - 1$ **(B)** $1 - e$ **(C)** $-\frac{1}{e}$ **(D)** $1 - \frac{1}{e}$ **(E)** $\frac{1}{e}$

11. $\int_0^1 xe^{x^2}\,dx =$

(A) $e - 1$ **(B)** $\frac{1}{2}(e - 1)$ **(C)** $2(e - 1)$ **(D)** $\frac{e}{2}$ **(E)** $\frac{e}{2} - 1$

12. $\int_0^{\pi/4} \sin 2\theta\, d\theta =$

(A) 2 **(B)** $\frac{1}{2}$ **(C)** -1 **(D)** $-\frac{1}{2}$ **(E)** -2

13. $\int_1^2 \frac{dz}{3 - z} =$

(A) $-\ln 2$ **(B)** $\frac{3}{4}$ **(C)** $2(\sqrt{2} - 1)$ **(D)** $\frac{1}{2}\ln 2$ **(E)** $\ln 2$

2. $\int_1^3 \frac{x}{x^2+1}\,dx =$

(A) $\ln 5$
(B) $\ln 10$
(C) $2 \ln 2$
(D) $\frac{1}{2}\ln 5$
(E) $\ln\frac{5}{2}$