AP Calculus AB
Test Review: Chapter 4

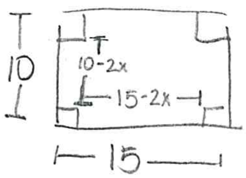
1. A rectangular sheet of cardboard measures 10 cm by 15 cm. Equal squares are cut out of each corner and the sides are turned up to form an open rectangular box. What is the maximum volume of the box?

Domain: $x > 0$ & $10 - 2x > 0$

$10 > 2x$

$x < 5$

$0 < x < 5$



$$V = x(10-2x)(15-2x)$$
$$= x(150 - 50x + 4x^2)$$
$$= 150x - 50x^2 + 4x^3$$

$$V' = 150 - 100x + 12x^2$$
$$= 2(6x^2 - 50x + 75)$$

$$V' = 0 \Rightarrow x = \frac{50 \pm \sqrt{2500 - 4(6)(75)}}{2(6)}$$

$$= \frac{50 \pm \sqrt{700}}{12} = 1.962,\ \cancel{6.371}$$

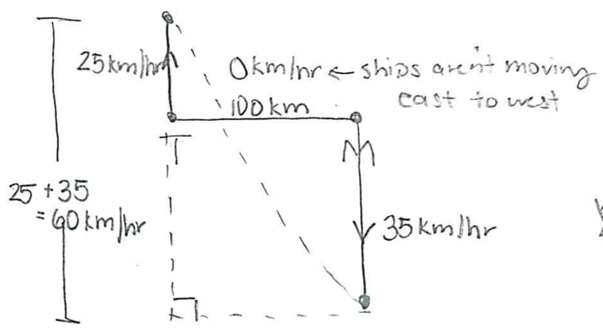
not in domain

V': + (1.962) max − (6.371) +

Volume maximized when $x = 1.962$

$\Rightarrow V(1.962) = 132.038\ cm^3$

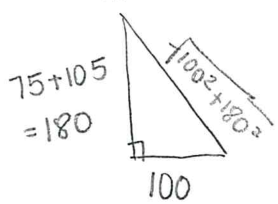
2. At 11:00 AM, Madison's ship is 100 km east of Tony's ship. Madison's ship is sailing south at 35 km/h and Tony's ship is sailing north at 25 km/h. How fast is the distance between the ships changing at 2:00 PM?



at 2 pm: (after 3 hrs)

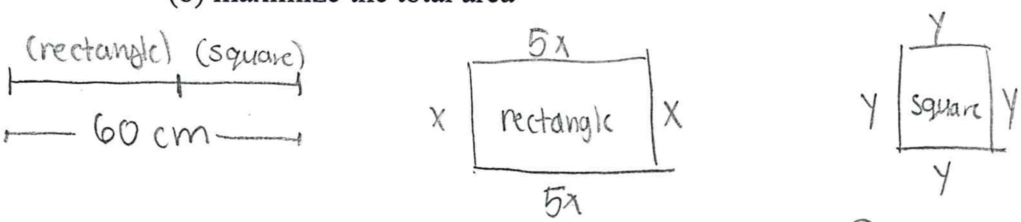
M: $\frac{25 km}{hr} \times 3hr = 75\ km$

T: $\frac{35 km}{hr} \times 3hr = 105\ km$



$$x^2 + y^2 = z^2$$
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$
$$100(0) + 180(60) = \sqrt{100^2 + 180^2}\,\frac{dz}{dt}$$
$$\frac{dz}{dt} = 52.449\ km/hr$$

3. A piece of wire 60 cm in length is cut into two pieces. The first piece forms a rectangle 5 times as wide as it is long. The second piece forms a square. Where should the wire be cut to:
(a) minimize the total area
(b) maximize the total area



$P_R = 5x + x + 5x + x = 12x$ $\quad P_S = 4y$ $\Rightarrow$ "helper equation" $12x + 4y = 60$

$\hookrightarrow 4y = 60 - 12x$

$y = 15 - 3x$

$\text{Area} = A_R + A_S = 5x^2 + y^2$

$= 5x^2 + (15-3x)^2 = 5x^2 + 225 - 90x + 9x^2$

$= 14x^2 - 90x + 225$

critical numbers:

$A' = 28x - 90$

A': − | + (Vmin at 3.2143)

endpoints:

- if all material used for rectangle $\Rightarrow 12x = 60$, $x = 5$
- if all material used for square $\Rightarrow x = 0$

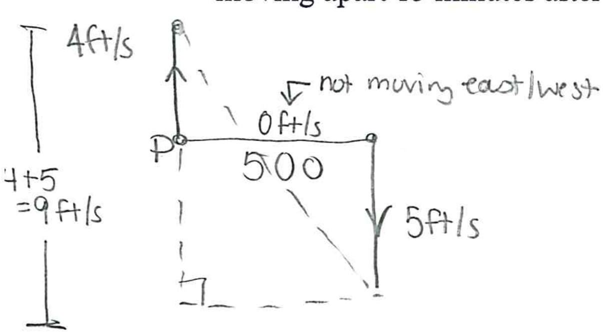
evaluate critical #s & endpoints:

$A(3.2143) = 80.357\ cm^3$ ← abs min

$A(0) = 225\ cm^3$ ← abs max

$A(5) = 125\ cm^3$

4. A man starts walking north at 4 ft/s from a point P. Five minutes later, a woman starts walking south at 5 ft/s from a point 500 ft due east of P. At what rate are the people moving apart 15 minutes after the woman starts walking?



$x^2 + y^2 = z^2$

$\cancel{2}x \frac{dx}{dt} + \cancel{2}y \frac{dy}{dt} = \cancel{2}z \frac{dz}{dt}$

$500(0) + 9300(9) = \sqrt{9300^2 + 500^2}\ \frac{dz}{dt}$

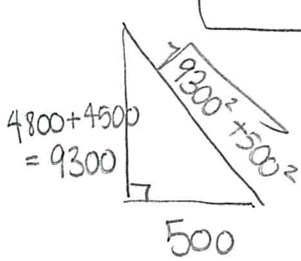
$\boxed{\frac{dz}{dt} = 8.987\ ft/s}$

man: 20 mins = 1200 sec

$\frac{4ft}{sec} \cdot 1200\ sec = 4800\ ft$

woman: 15 mins = 900 sec

$\frac{5ft}{sec} \cdot 900\ sec = 4500\ ft$



5. Consider the function $f(x) = \begin{cases} -x & \text{if } x \le 1 \\ x-2 & \text{if } x > 1 \end{cases}$. Note that $f(0) = f(2) = 0$. Your friend claims that by the Mean Value Theorem, $f'(c)$ should be zero for some c with $0 < c < 2$.

(a) Find such a value of c or show that it does not exist.

$f'(x) = \begin{cases} -1, & \text{if } x \le 1 \\ 1, & \text{if } x > 1 \end{cases}$

$\therefore$ no c exists

(b) Does your answer to part (a) contradict the Mean Value Theorem? Give reasons for your answer.

no contradiction b/c f is not differentiable on the interval. We cannot even think of applying MVT until we've guarenteed our function is continuous & differentiable!

6. Find all numbers c that satisfy the conclusion of the Mean Value Theorem for the function $f(x) = x^3 + x - 1$ on the closed interval [0,2].

$f'(x) = 3x^2 + 1$

slope: $\frac{f(2) - f(0)}{2 - 0} = \frac{9-(-1)}{2} = 5$

$3c^2 + 1 = 5$

$3c^2 = 4$

$c^2 = 4/3$

$c = \cancel{\pm} \, 2/\sqrt{3}$

not within interval

Since our function is continuous & differentiable, we can apply the MVT

$\Rightarrow \boxed{c = 2/\sqrt{3}}$

7. If the graph of $y = ax^3 - 6x^2 + bx - 4$ has a point of inflection at (2,-2), what is the value of $a + b$?

$\hookrightarrow$ f'' changes sign (to change sign, must hit 0)

$y' = 3ax^2 - 12x + b - 4$

$y'' = 6ax - 12$

$y''|_{(2,-2)} = 0 \Rightarrow 6ax - 12 = 0$

$6a(2) - 12 = 0$

$12a = 12$

$a = 1$

(2,-2) is a point on the curve!

$y = ax^3 - 6x^2 + bx - 4$

$-2 = a(2)^3 - 6(2)^2 + b(2) - 4$

$-2 = 8 - 24 + 2b - 4$

$2b = 18$

$b = 9$

$a + b = 1 + 9 = \underline{\underline{10}}$