Chapter 5 Review Station 1 – Definite Integrals Algebraically

Evaluate each of the following definite integrals algebraically.

1. $\int_1^5 (x^2 - 2x + 1)dx$

$$\frac{x^3}{3} - x^2 + x \Big|_1^5 = \left(\frac{125}{3} - 25 + 5\right) - \left(\frac{1}{3} - 1 + 1\right)$$
$$= \frac{124}{3} - 20$$
$$\boxed{= \frac{64}{3}}$$

2. $\int_4^9 (\sqrt{x} - \frac{1}{x^2})dx$

$$\int_4^9 (x^{\frac{1}{2}} - x^{-2})dx$$
$$= \frac{2}{3}x^{3/2} + \frac{1}{x} \Big|_4^9$$
$$= \left(\frac{54}{3} + \frac{1}{9}\right) - \left(\frac{16}{3} + \frac{1}{4}\right)$$
$$= \frac{38}{3} + \frac{1}{9} - \frac{1}{4}$$
$$= \boxed{\frac{451}{36} \approx 12.527}$$

3. $\int_{-\pi/2}^{\pi/2} (\sin x + 1)dx$

$$= -\cos x + x \Big|_{-\frac{\pi}{2}}^{\pi/2}$$
$$= \left(0 + \frac{\pi}{2}\right) - \left(0 - \frac{\pi}{2}\right)$$
$$\boxed{= \pi}$$

4. $\int_1^e e^x - \frac{1}{x}dx$

$$= e^x - \ln|x| \Big|_1^e$$
$$= (e^e - \ln e) - (e - \ln 1)$$
$$= e^e - 1 - e + 0$$
$$\boxed{\approx 11.4359}$$

5. $\int_1^2 \frac{1-x-x^2}{x^3}dx$

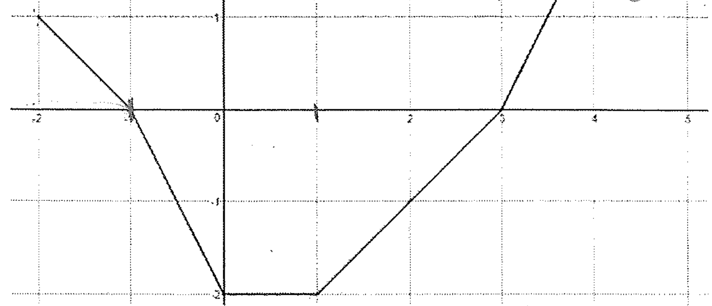
$$\int_1^2 \left(\frac{1}{x^3} - \frac{1}{x^2} - \frac{1}{x}\right)dx$$
$$= -\frac{1}{2}x^{-2} + \frac{1}{x} - \ln|x| \Big|_1^2$$
$$= \left[\left(-\frac{1}{2}\right)\left(\frac{1}{4}\right) + \frac{1}{2} - \ln 2\right] - \left[\left(-\frac{1}{2}\right)(1) + 1 - \ln(1)\right]$$
$$= -\frac{1}{8} + \cancel{\frac{1}{2}} - \ln 2 + \cancel{\frac{1}{2}} - \cancel{1} + 0$$
$$\boxed{= -\frac{1}{8} - \ln 2 \approx -.8181}$$

Chapter 5 Review Station 2 – Graphical Interpretation of the Definite Integral

Use the graph of f to the right and that $g(x) = \int_{-1}^{x} f(t)dt$

[g: $-\frac{1}{2}$ 0 -1 -3 -4.5 -5 -4 -2

[g'' 1 -1 -2 0 1 1 2 1 0



1. Find $g(1), g'(1)$ and $g''(1)$

$g(1)=-3$ $g'(1)=-2$ $g''(1)=$ DNE

2. Find $g(2), g'(2)$ and $g''(2)$

$g(2)=-4.5$; $g'(2)=-1$; $g''(2)=1$

3. Find $g(-2), g'(-2)$ and $g''(-2)$

$g(-2)=-\frac{1}{2}$; $g'(-2)=1$; $g''(-2)=-1$

$g(x)=\int_{-1}^{x} f(t)dt$

$g'(x)=f(x)$

$g''(x)=f'(x)$

4. On what intervals is g increasing? Decreasing? Concave up/down? Justify.

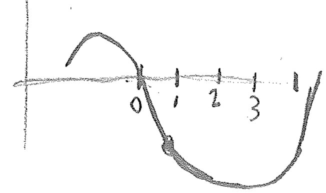
INC: $(-2,-1)$, $(3,5)$ DEC: $(-1,5)$ CONC UP: $(1,4)$ DER OF GRAPH POS

CONC DWN: $(-2,0)$ DER OF GRAPH NEG

5. Local Extrema? Points of Inflection? Justify.

ABS MAX $(-1,0)$ ABS MIN $(3,-5)$

POI: $(1,-3)$?



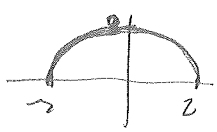
6. Find the average value of f on [-2,3].

$\frac{1}{5}\int_{-2}^{3} f(t)dt = \frac{1}{5}[g(3)-g(-2)] = \left(\frac{1}{5}\right)\left(\frac{-9}{2}\right)$

$\frac{1}{5}[-5-(-\frac{1}{2})]$ $\boxed{= \frac{-9}{10}}$

Using geometry to evaluate the definite integrals.

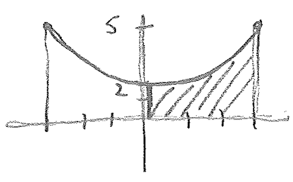
7. $\int_{-2}^{2} \sqrt{4-x^2}dx$



$A=\left(\frac{1}{2}\right)\pi(2^2)$

$\boxed{=2\pi}$

8. $\int_{0}^{3} (-\sqrt{9-x^2}+5)dx$



$A=15-[\frac{1}{4}\cdot\pi\cdot 9]$

$\boxed{=15-\frac{9\pi}{4}}$

9. $\int_{-1}^{3} (10-2x)dx$

≈ 7.9314

Chapter 5 Review Station 3 – Approximations

1. As a pot of coffee cools down, the temperature of the coffee is modeled by a decreasing differentiable function C for $0 \le t \le 12$, where time is measured in minutes and the temperature $C(t)$ is measured in degrees Celsius. Selected values of time t are shown in the table below.

t	0	3	5	7	8	12
$C(t)$	65	57	50	46	44	40

a. Evaluate $\int_0^{12} C'(t)dt$. Using correct units, explain the meaning of your answer in the context of this problem.

$-25^{o}C$ THE CHANGE IN TEMP FROM $t=0$ TO $t=12$

b. Using correct units, explain the meaning of $\frac{1}{12}\int_0^{12} C(t)dt$ in the context of this problem. Use a trapezoidal sum with 5 subintervals indicated by the table to approximate $\frac{1}{12}\int_0^{12} C(t)dt$.

AVG TEMP OF THE COFFEE OVER THE 12 MINUTES

$T_A = \frac{3}{2}[65+57] + [57+50] + [50+46] + \frac{1}{2}[46+44] + \frac{4}{2}[44+40]$

$= 183 + 107 + 96 + 45 + 168 = 599$

$\frac{1}{12}[599] = 49.9166$

c. Must there be a time on [8,12] such that temperature is decreasing at a rate of 1 degree Celsius per minute? Explain.

$(8, 44)$ $(12, 40)$

$m_{8-12} = \frac{40-44}{12-8} = \frac{-4}{4} = -1$

BY MVT, YES, THE AVG RATE OF CHANGE ON [8,12] IS −1 AND TEMP IS CONT

d. For $12 \le t \le 15$, the rate of cooling is modeled by $C'(t) = -2\cos t$. Based on the model, what is the temperature of the coffee at time $t = 15$?

$\int_{12}^{15} -2\cos t \, dt$

$= -2\sin t \, dt$

$= -2\sin 15 + 2\sin 12$

$= -2.3737$

$T_{12} = 40 - 2.3737$

$T_{12} = 37.6263^{o}C$

2. Using the programs on your calculator, estimate each of the following definite integrals using n = 10 subintervals.

Integral	RRAM	LRAM	MRAM	Trap	Simpson's	Exact
$\int_{-1}^{2} x^2 \cos x \, dx$	.0244866127	.6859535083	.4120833266	.3552200605	.3929351881	.3931411344
$\int_1^4 \frac{e^{\sqrt{x}} - 2x + 1}{2 + x} dx$	.6236536327	.7760290106	.6957011262	.6998413216	.6970873384	.697080801

Chapter 5 Review Stations – Applications

1. (no calculator) A particle moves along the $x-axis$ on the interval $0 \le t \le \frac{5\pi}{4}$. The velocity of the particle is given by $v(t) = 2\sin t$. The particle is at position $x = -4$ $when\ t = 0$.

   NO CALC a. For $0 \le t \le \frac{5\pi}{4}$, when is the particle moving to the right?

$(0, \pi)$

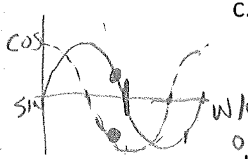
   NO CALC b. Find the total distance traveled by the particle during $0 \le t \le \frac{5\pi}{4}$.

$\int_0^{\pi} 2\sin t\,dt - \int_{\pi}^{\frac{5\pi}{4}} 2\sin t\,dt = -2\cos t\Big|_0^{\pi} + 2\cos t\Big|_{\pi}^{\frac{5\pi}{4}}$

$= [(-2)(-1)] - [(-2)(1)] + [(2)(-\frac{\sqrt{2}}{2}) - (2)(-1)]$

$(2+2) + (-\sqrt{2}+2) = 6 - \sqrt{2}$ UNITS

   c. Find the acceleration of the particle at t = 3. Is the particle speeding up or slowing down at t = 3? Justify.



$a(t) = 2\cos t$ W/O CALC ONLY KNOW $a(3) = 2\cos(3)$ "NEG"

$v(t) = 2\sin t$ W/O CALC ONLY KNOW "POS"

SO VELOCITY IS POS BUT DEC SO SLOWING DOWN

   d. Find the position of the particle at t = 3.

$s(t) = -2\cos t + C$

$s(0) = -4$

$-4 = -2\cos(0) + C$

$-4 = -2 + C$

$-2 = C$

$s(t) = -2\cos t - 2$

$s(3) = -2\cos(3) - 2$

2. (calculator) On a typical day, the snow on Pike Mountain melts at a rate modeled by the function $M(t) = \frac{\pi}{6}\sin\frac{\pi t}{12}$. A snow maker adds snow at a rate modeled by the function $S(t) = 0.006t^2 - 0.12t + 0.87$. Both M and S have units in inches per hour and t is measured in hours for $0 \le t \le 6$. At time t = 0, the mountain has 40 inches of snow.

   a. How much snow will melt during the 6 hour period? Indicate units of measure.

$\frac{\pi}{6}\int_0^6 \sin\frac{\pi t}{12}\,dt = 2$ INCHES

   b. Write an expression for $I(t)$, the total number of inches of snow at any time t.

$I(t) = 40 + \int_0^t \underbrace{(.006t^2 - .12t + .87)}_{S(t)}dt - \int_0^t \underbrace{\frac{\pi}{6}\sin\frac{\pi t}{12}}_{M(t)}\,dt$

   c. Find the rate of change of the total amount of snow at time t = 3.

$\frac{d}{dt}(I(t)) = 0 + S(3) - M(3) = 0 + .564 - .3702 = .1938$ IN/HR

   d. For $0 \le t \le 6$, at what time is the amount of snow a maximum? What is the maximum amount of snow? Justify.

@ $t = 4.2405$ HR

$I'(t)$ IS POS $[0, 4.2405)$ AND NEG $(4.2405, 6]$

$I(4.2405) = 41.6518$ INCHES

3. Let $f(x) = \begin{cases} -x^2 + 4x, & 1 \le x < 4 \\ x - 2\sqrt{x}, & x \ge 4 \end{cases}$. Find the average value of f on the interval [1, 9].

$\frac{1}{8}\left[\int_1^4 (-x^2 + 4x)\,dx + \int_4^9 (x - 2x^{\frac{1}{2}})\,dx\right]$

$= \frac{1}{8}[9 + 7.1667]$

$= 2.0208$