Sample Syllabus 1 Contents

Curricular Requirements

CR1a The course is structured around the enduring understandings within Big Idea 1: Limits.
- See page 1

CR1b The course is structured around the enduring understandings within Big Idea 2: Derivatives.
- See pages 1, 3

CR1c The course is structured around the enduring understandings within Big Idea 3: Integrals and the Fundamental Theorem of Calculus.
- See page 3

CR2a The course provides opportunities for students to reason with definitions and theorems.
- See page 1

CR2b The course provides opportunities for students to connect concepts and processes.
- See page 2

CR2c The course provides opportunities for students to implement algebraic/computational processes.
- See page 2

CR2d The course provides opportunities for students to engage with graphical, numerical, analytical, and verbal representations and demonstrate connections among them.
- See pages 2, 3, 4

CR2e The course provides opportunities for students to build notational fluency.
- See page 4

CR2f The course provides opportunities for students to communicate mathematical ideas in words, both orally and in writing.
- See pages 3, 4

CR3a Students have access to graphing calculators.
- See page 1

CR3b Students have opportunities to use calculators to solve problems.
- See page 1

CR3c Students have opportunities to use a graphing calculator to explore and interpret calculus concepts.
- See page 2

CR4 Students and teachers have access to a college-level calculus textbook.
- See page 4

Advanced Placement Calculus AB

The overall goal of this course is to help students understand and apply the three big ideas of AB Calculus: limits, derivatives, and integrals and the Fundamental Theorem of Calculus. Imbedded throughout the big ideas are the mathematical practices for AP Calculus: reasoning with definitions and theorems, connecting concepts, implementing algebraic/computational processes, connecting multiple representations, building notational fluency, and communicating mathematics orally and in well-written sentences. All students are required to complete summer work reviewing precalculus and Algebra 2 concepts prior to entry in the course. Students will be provided with and expected to regularly use a school issued TI-Nspire CAS CX graphing calculator. **[CR3a]**

[CR3a] — Students have access to graphing calculators.

Course Outline

Limits and Continuity (4 weeks) [CR1a]

[CR1a] — The course is structured around the enduring understandings within Big Idea 1: Limits.

Students will express limits symbolically using correct notation and evaluate limits expressed symbolically.

- Sample activity: Limits of the Hand - Students will trace their hand onto grid paper with squares of various sizes. They are to explore the relationship between the grid size, the area of the hand, and the upper and lower limits of the area.

Students will estimate and determine limits of functions. Students will understand behavior of functions using limits.

- Sample activity: Students use the calculator to estimate limits of functions that have asymptotes. **[CR3b]**

[CR3b] — Students have opportunities to use calculators to solve problems.

Students will analyze functions for intervals of continuity.

- Sample activity: Part one of a graphing calculator activity from Texas Instruments's *Calculus: Continuity and Differentiability* allows students to explore piecewise graphs and determine conditions for continuity.

Students will determine the applicability of important calculus theorems involving continuity.

- Sample activity: Students will work in groups to complete *Joys of the Intermediate Value Theorem* by James Tanton to gain a deeper understanding of the application possibilities of the Intermediate Value Theorem. **[CR2a]**

[CR2a] — The course provides opportunities for students to reason with definitions and theorems.

Derivatives (10-12 weeks) [CR1b: derivatives]

[CR1b] — The course is structured around the enduring understandings within Big Idea 2: Derivatives.

Students will compute the derivative of a function as the limit of a difference quotient.

- Sample activity: Students will work in groups to determine how fast someone in their group can run half the length of the soccer field. Students will discover that the limit of the average velocity as the length of the time intervals approaches zero is the instantaneous velocity.

Students will estimate and calculate derivatives.

- Sample activity: The graphing calculator activity *The Derivative of a Polynomial and the Discovery of the Power Rule* will walk students through investigations of the value of the slope of a tangent line to polynomial curves at various points. A scatter plot will be generated so that the students will discover that the derivatives themselves are in fact functions. The graphing calculator activity *Move Those Chains* allows students to explore the chain rule. Students are asked to make a conjecture about the formula for the derivative of a given function based on the power rule. They are then asked to graph their derivative function and compare it to the graph of $f'(x)$. Next, they will examine statements about various derivatives of composite functions. They will observe patterns and try to create a rule for finding the derivative of other composite functions. **[CR2d: connection between analytical and graphical] [CR3c]**
- Sample activity: A worksheet gives students a table of values for f, f', g, and $g'$and asks the students to evaluate expressions such as the derivative of $f\ (g(x))$ at $x = 3$. **[CR2d: numerical]**

[CR2d] — The course provides opportunities for students to engage with graphical, numerical, analytical, and verbal representations and demonstrate connections among them.

[CR3c] — Students have opportunities to use a graphing calculator to explore and interpret calculus concepts.

Students will determine higher order derivatives and use derivatives to analyze properties of a function.

- Sample activity: Students will work in groups to sketch a graph of f' given the graph of f. Then they will be given the graph of $f'$and asked to sketch a possible graph of f. The graphing calculator activity *Critical Points and Local Extrema* leads students to understand the connections between the critical points and local extrema.

Students will recognize the connection between differentiability and continuity.

- Sample activity: Part 2 of the graphing calculator activity *Calculus: Continuity and Differentiability* allows students to explore graphs of piecewise functions in an attempt to discover whether differentiability is necessary or sufficient to guarantee continuity and whether continuity is necessary or sufficient to guarantee differentiability. **[CR2b]**

[CR2b] — The course provides opportunities for students to connect concepts and processes.

Students will interpret the meaning of a derivative within a real-world problem.

- Sample activity: We revisit the running activity to help students understand that the derivative of a function is the same as its instantaneous rate of change.

Students will solve problems involving the slope of a tangent.

- Sample activity: Students choose a function and graph it in their calculator. Students then pick a point on their graph. They need to find the equation of the tangent line for that point and graph it in the same window as their original. Students keep zooming in to their chosen point until they are able to recognize the relationship between the tangency and local linearity.

Students will solve problems involving indeterminate forms.

- Sample activity: Students take a quiz that asks them to determine if L'Hospital's Rule applies to a given limit and then apply the rule to evaluate the limit analytically, if appropriate. **[CR2c]**

[CR2c] — The course provides opportunities for students to implement algebraic/computational processes.

Students will solve problems involving related rates, optimization, and rectilinear motion.

- Sample activity: Students work with a partner on a set of problems involving the *Misadventures of Gale the Clown*. The story about Gale involves a variety of related rates problems that require students to solve an equation analytically and write a coherent explanation of their thinking. **[CR2d: verbal]**

[CR2d] — The course provides opportunities for students to engage with graphical, numerical, analytical, and verbal representations and demonstrate connections among them.

Students will verify and estimate solutions to differential equations.

- Sample activity: Students will work together in groups to sketch slope fields when given differential equations in analytic form. Then students will sketch possible solutions through given points. The class will compare and discuss their different possible solutions. Graphing calculators will be used at the conclusion to verify results. Students will discuss differences in their solutions and the solutions on the calculator and analyze those differences. **[CR2f: oral]**

[CR2f] — The course provides opportunities for students to communicate mathematical ideas in words, both orally and in writing.

Students will apply the Mean Value Theorem to describe the behavior of a function over an interval. **[CR1b: Mean Value Theorem]**

- Sample activity: The graphing calculator activity *MVT for Derivatives* allows students to relate the average rate of change of a function to an instantaneous rate of change. A follow-up discussion revisiting the running activity can help solidify the concept.
- Sample activity: Students work individually on a worksheet that asks them to decide if the hypotheses and conclusion of the Mean Value Theorem apply to a variety of functions given analytically. The students get together in pairs to compare their answers before having a class discussion. **[CR2d: analytical]**

[CR1b] — The course is structured around the enduring understandings within Big Idea 2: Derivatives.

[CR2d] — The course provides opportunities for students to engage with graphical, numerical, analytical, and verbal representations and demonstrate connections among them.

Integrals and the Fundamental Theorem of Calculus (10-12 weeks) [CR1c]

[CR1c] — The course is structured around the enduring understandings within Big Idea 3: Integrals and the Fundamental Theorem of Calculus.

Students will recognize antiderivatives of basic functions.

- Sample activity: Students create a slope field for $dy/dx = \sin x$ and are asked what function the slope field appears to show. This can lead to a discussion of the constant of integration. Students create and test antiderivative rules for various basic functions.

Students will interpret the definite integral as the limit of a Riemann sum, express the limit of a Riemann sum in integral notation, and approximate a definite integral using a variety of methods.

- Sample activity: The graphing calculator activity *Riemann Sums* allows students to see different Riemann sums expressed graphically and to use the values of these sums to estimate the area of a region using left-hand endpoint, right-hand endpoint, and midpoint methods. They can then see the relationship between the exact area and the limit of the Riemann sum.
- Sample activity: Limits of various Riemann sums are written on the board and students are asked to rewrite each as a definite integral.

Students will calculate a definite integral using areas and properties of definite integrals.

- Sample activity: Students are asked on a worksheet how far they travel if they drive at 60 mph for 2 hours. They are then asked to create a velocity graph representing this situation. Students then evaluate the integral of $f(t) = 60$ from 0 to 2 and discuss any connections they see. Students will analyze functions defined by an integral using both parts of the Fundamental Theorem of Calculus. Part one states if f is continuous on $[a, b]$, then the functions g defined by $g(x) = \int_a^x f(t)dt$ is an antiderivative of f. That is, $g'(x) = f(x)$ for $a < x < b$. Part two states if f is continuous on $[a, b]$, then $\int_a^b f(x)dx = F(b) - F(a)$ where $F(x)$ is any antiderivative of $f(x)$. **[CR2e]**
- Sample activity: Students will complete a write-pair-share activity which requires them to write a paragraph using well-written sentences about what the FTC means in the context of a given application problem. **[CR2f: written]** Students will calculate antiderivatives and definite integrals.
- Sample activity: Students complete the graphing calculator activity *Integration by Substitution* to explore methods for computing integrals of functions that are not in one of the standard forms.

[CR2e] — The course provides opportunities for students to build notational fluency.

[CR2f] — The course provides opportunities for students to communicate mathematical ideas in words, both orally and in writing.

Students will interpret the meaning of a definite integral within a problem and use the definite integral to solve problems in various contexts, including those involving average value, motion, area, and volume.

- Sample activity: A volume of revolution project requires students to determine the volume of two different shapes. One shape they create on their own from an equation of their choice revolved about an axis. They must create a three-dimensional model and cross-section of the shape. For their second shape, they pick a three-dimensional shape and model a cross-section. Students must determine the limits of integration and axis of revolution. They must write and use an integral to find the volume of both shapes.

Students will interpret, create, and solve differential equations from problems in context and analyze differential equations to obtain general and specific solutions.

- Sample activity: Students work in groups. Each group is given a collection of application problems involving initial conditions. After the groups are finished, the class will discuss and analyze various techniques each group used in solving the problems. **[CR2d: verbal]**

[CR2d] — The course provides opportunities for students to engage with graphical, numerical, analytical, and verbal representations and demonstrate connections among them.

Additional Information

Throughout the course, students will be responsible for completing homework and in-class activities for 20 percent of their grade. Assessments such as quizzes, tests, and projects will count for 80 percent of their grade. Students will solve a variety of multiple-choice and free-response questions over limits, derivatives, and integrals from past AP exams. They will justify their answers using calculus-based concepts and present their reasoning orally at the board several times throughout the school year. **[CR2f: oral]** In addition, they will submit reflective journal entries about assigned free-response AP questions before the AP exam in May.

[CR2f] — The course provides opportunities for students to communicate mathematical ideas in words, both orally and in writing.

Texts and Resources

Stewart, James. *Single Variable Calculus: Early Transcendentals AP Edition*. 6th ed. Bellmont, CA: Brooks/Cole, 2010. **[CR4]**

Finney, Ross L., Franklin D. Demana, Bet K. Waits, and Daniel Kennedy. *Calculus: Graphical, Numerical, Algebraic.* 3rd ed. Boston: Prentice Hall, 2010.

[CR4] — Students and teachers have access to a college-level calculus textbook.

Graphing calculator activities can be found at http://education.ti.com/en/timathnspired/us/calculus.