Example 1:

Find the general antiderivatives of each of the following using you knowledge of how to find derivatives.

a) $f(x) = 2x$ b) $f'(x) = x$ c) $F'(x) = \frac{2}{3}x^{\frac{4}{7}}$ d) $g'(x) = \frac{1}{x^2}$ e) $\frac{dy}{dx} = \cos x$

Example 2:

Find all functions g such that $g'(x) = 4\sin x + \frac{2x^4 - \sqrt{x} + x}{x}$.

Example 3:

Solve the differential equation $f'(x) = 3x^2$ if $f(2) = -3$. Find both the general and particular solutions.

Example 4:

Find the particular solution to the following differential equation if $\frac{dy}{dx} = e^x + 20\left(1+x^2\right)^{-1}$ and $y(0) = -2$.

Example 5:

Find the particular solution to the following differential equation if $\frac{d^2y}{dx^2} = 12x^2 + 6x - 4$ and

a) $y'(1) = 3$ and $y(0) = -6$

b) $y(0) = 4$ and $y(1) = 1$.

Example 6:

a) Evaluate $\int \frac{\sin x}{\cos^2 x}dx$

b) Evaluate $\int \left(\tan^2 p + 4\right)dp$

Show all work. No Calculator

Multiple Choice

1. If $f'(x) = 12x^2 - 6x + 1$, $f(1) = 5$, then $f(0)$ equals

(A) 2 (B) 3 (C) 4 (D) –1 (E) 0

2. Find all functions g such that $g'(x) = \dfrac{5x^2 + 4x + 5}{\sqrt{x}}$

(A) $g(x) = 2\sqrt{x}\left(x^2 + \frac{4}{3}x - 5\right) + C$ (B) $g(x) = 2\sqrt{x}\left(x^2 + \frac{4}{3}x + 5\right) + C$

(C) $g(x) = 2\sqrt{x}\left(5x^2 + 4x - 5\right) + C$ (D) $g(x) = \sqrt{x}\left(x^2 + \frac{4}{3}x + 5\right) + C$

(E) $g(x) = \sqrt{x}\left(5x^2 + 4x + 5\right) + C$

3. Determine $f(t)$ when $f''(t) = 2(3t+1)$ and $f'(1) = 3$, $f(1) = 5$.

(A) $f(t) = 3t^3 - 2t^2 + 2t + 2$ (B) $f(t) = t^3 - 2t^2 + 2t + 4$

(C) $f(t) = 3t^3 + t^2 - 2t + 3$ (D) $f(t) = t^3 - t^2 + 2t + 3$

(E) $f(t) = t^3 + t^2 - 2t + 5$

4. Consider the following functions:

I. $F_1(x) = \dfrac{\sin^2 x}{2}$

II. $F_2(x) = -\dfrac{\cos 2x}{4}$

III. $F_3(x) = -\dfrac{\cos^2 x}{2}$

Which are antiderivatives of $f(x) = \sin x \cos x$? (Hint: take the derivative of each and manipulate)

(A) II only (B) I only (C) I & III only (D) I, II, & III (E) I & II only

5. A particle moves along the x-axis so that its acceleration at time t is $a(t) = 8 - 8t$ in units of feet and seconds. If the velocity of the particle at $t = 0$ is 12 ft/sec, how many seconds will it take for the particle to reach its furthest point to the right?

(A) 6 seconds (B) 5 seconds (C) 3 seconds (D) 7 seconds (E) 4 seconds

Free Response

6. Evaluate the following:

(a) $\int\left(\sqrt{x^3} + 2x + 1\right)dx$

(b) $\int\left(\dfrac{x^3 + 2x - 3}{x^4}\right)dx$

(c) $\int\left(2t^2 - 1\right)^2 dt$

(d) $\int\left(\theta^2 + \sec^2\theta - \csc\theta\cot\theta\right)d\theta$

(e) $\int\left(\dfrac{\cos x}{1 - \cos^2 x}\right)dx$

(f) $\int\left(\cos x + 3^x\right)dx$

7. Solve the following differential equations. Find the general solution, then find the particular solution using the initial condition.

(a) $f'(x) = 4x$, $f(0) = 6$ (b) $h'(t) = 8t^3 + 5$, $h(1) = -4$ (c) $f''(x) = 2$, $f'(2) = 5$, $f(2) = 10$

(d) $f''(x) = x^{-3/2}$, $f'(4) = 2$, $f(0) = 0$ (e) $f''(x) = \sin x$, $f'(0) = 1$, $f(0) = 6$

ANTIDERIVATIVES & INITIAL VALUE PROBLEMS PRACTICE

1) $\int \pi^3 \, dx =$

A) $3\pi^2 x + c$
B) 0
C) $\pi^3 x + c$
D) $3\pi^2 + c$
E) $\frac{\pi^4}{4} + c$

2) $\int \left(x^4 - x^3 + x^2\right) dx =$

A) $\frac{x^5}{4} - \frac{x^4}{3} + \frac{x^3}{2} + c$
B) $5x^5 - 4x^4 + 3x^3 + c$
C) $\frac{x^5}{5} - 3x^2 + \frac{x^3}{3} + c$
D) $4x^3 - 3x^2 + 2x + c$
E) $\frac{x^5}{5} - \frac{x^4}{4} + \frac{x^3}{3} + c$

3) $\int \left(x^2 + 2\right)\left(1 - x\right) dx =$

A) $\frac{x^3}{3} - 2x^2 + c$
B) $-\frac{x^4}{4} + \frac{x^3}{3} - x^2 + c$
C) $-3x^2 + 2x - 2 + c$
D) $-\frac{x^4}{4} + \frac{x^3}{3} - x^2 + 2x + c$
E) $\left(\frac{x^3}{3} - 2x\right)(x - x^2) + c$

4) $\int \left(x^2 - 2\right)^2 dx =$

A) $\left(\frac{x^2 - 2}{3}\right)^3 + c$
B) $\frac{(x^2 - 2)^3}{6x} + c$
C) $\frac{2x}{3}(x^2 - 2)^3 + c$
D) $\left(\frac{x^3}{3} - 2x\right)^2 + c$
E) $\frac{x^5}{5} - \frac{4x^3}{3} + 4x + c$

5) $\int x^3 \left(x^3 - 2\right) dx =$

A) $\frac{x^7}{7} - \frac{x^4}{2} + c$
B) $\frac{x^4}{4}\left(\frac{x^4}{4} - 2x\right) + c$
C) $\frac{3(x^3 - 3)^2}{2} + c$
D) $3x^2\left(\frac{x^4}{4} - 2x\right) + c$
E) $6x^5 - 6x^2 + c$

6) $\int \frac{3x^5 + 2x^3 - x^2}{x^2} dx =$

A) $\frac{x^6 + x^4 - x^3}{6x^3} + c$
B) $\frac{15x^4 + 6x^2 - 2x}{2x} + c$
C) $18x^6 + 8x^2 - 2x + c$
D) $3x^4 + 2x^2 - x + c$
E) $\frac{3}{4}x^4 + x^2 - x + c$

7) $\int \frac{1}{\sqrt{x}}\,dx =$

A) $\frac{1}{2}x\sqrt{x} + c$

B) $\frac{2}{3}x\sqrt{x} + c$

C) $-\frac{1}{2}x\sqrt{x} + c$

D) $2\sqrt{x} + c$

E) $\frac{1}{2}\sqrt{x} + c$

8) $\int \frac{1}{\sqrt[3]{x^2}}\,dx =$

A) $\frac{3}{2}x^{\frac{2}{3}} + c$

B) $-\frac{1}{3}x^{-\frac{1}{3}} + c$

C) $3x^{\frac{1}{3}} + c$

D) $\frac{1}{3}x^{\frac{1}{3}} + c$

E) $-3x^{-\frac{1}{3}} + c$

9) $\int \left(\frac{3}{u^{\frac{3}{4}}} - 4\sqrt[3]{u} + 1 \right) du =$

A) $\frac{21}{4u^{\frac{7}{4}}} - \frac{1}{3}u^{\frac{4}{3}} + u + c$

B) $\frac{3}{4}u^{\frac{1}{4}} - \frac{16}{3}u^{\frac{4}{3}} + u + c$

C) $12u^{\frac{3}{4}} - 3u^{\frac{5}{3}} + u + c$

D) $\frac{4}{3}u^{\frac{1}{4}} - \frac{3}{16}u^{\frac{4}{3}} + u + c$

E) $12u^{\frac{1}{4}} - 3u^{\frac{4}{3}} + u + c$

10) $\int \left(\sqrt[4]{x^3} - \frac{2}{\sqrt[3]{x^2}} \right) dx =$

A) $\frac{3}{4\sqrt[4]{x}} - \frac{4}{3\sqrt[3]{x}} + c$

B) $\frac{4}{7}\sqrt[4]{x^3} - \frac{2}{3}\sqrt[3]{x} + c$

C) $\frac{4}{7}x\sqrt[4]{x^3} - 6\sqrt[3]{x} + c$

D) $\frac{4}{7}x\sqrt[4]{x^3} - \frac{6}{5}x\sqrt[3]{x^2} + c$

E) $\frac{7}{4}x\sqrt[4]{x^3} - \frac{5}{6}x\sqrt[3]{x^2} + c$

11) If $g'(x) = 4x^3 + 3x^2 + 6x$ and $g(1) = -3$, then $g(x) =$

A) $x^4 + x^3 + 6x^2 - 11$

B) $\frac{x^4}{4} + \frac{x^3}{3} + 3x^2 - \frac{79}{12}$

C) $12x^2 + 6x + 6$

D) $x^4 + x^3 + 3x^2$

E) $x^4 + x^3 + 3x^2 - 8$

12) Which of the following defines a function f such that $f'(x) = \sqrt{x}$ and the graph of function f pass through the point (9,0)?

A) $f(x) = \frac{2}{3}x\sqrt{x} - 18$

B) $f(x) = x\sqrt{x} - 3x$

C) $f(x) = \frac{x\sqrt{x}}{3} + 9$

D) $f(x) = \frac{1}{2}\sqrt{x} - 3$

E) $f(x) = \frac{3}{2}x\sqrt{x} - 18$

13) The slope of a curve at each point (x,y) is given by $4x - 1$. Which of the following is an equation for this curve if it passes through the point (-2,3)?

A) $y = 2x^2 - x - 7$
B) $y = 4x^2 - x - 15$
C) $y = 2x^2 - x + 7$
D) $y = x^2 - 4x - 9$
E) $y = 2x^2 - x$

14) If function f has a derivative defined by $f'(x) = \frac{x+1}{\sqrt{x}}$ and $f(1) = 0$, then $f(4) =$

A) $\frac{20}{3}$
B) $-\frac{4}{3}$
C) $\frac{4}{3}$
D) $-\frac{8}{3}$
E) $\frac{3}{4}$

15) The slope of the line tangent to the graph of a function f at any point (x,y) is given by $x^3 - x$. If the graph of function f passes through the point (2,1), find f(0).

A) 1
B) 2
C) 3
D) -1
E) 0

16) A function f has a derivative $f'(x) = 3 - 2x$. An equation of the line tangent to the graph of function f at $x = 2$ is $y - 7 = -(x - 2)$. What is an equation of function f?

A) $f(x) = -x^2 + 3x$
B) $f(x) = -3x^2 + x - 3$
C) $f(x) = -x^2 + 3x - 3$
D) $f(x) = 3x^2 + 3x - 1$
E) $f(x) = x^2 - 3x + 3$

17) If $h''(x) = x - 2$, $h'(4) = 0$, and $h(0) = 4$, then $h(x) =$

A) $\frac{x^3}{6} - x^2 + 4$
B) $2x^3 - x^2 + 4$
C) $\frac{x^3}{3} - \frac{x^2}{2} + 4$
D) $\frac{x^3}{6} - x^2$
E) $\frac{1}{2}x^2 + 2x + 4$

18) At each point (x,y) on a curve, $\frac{d^2y}{dx^2} = 6x$. Additionally, the line $y = 6x + 4$ is tangent to the curve at $x = -2$. Which of the following is an equation of the curve that satisfies these conditions?

A) $y = 6x^2 - 32$
B) $y = x^3 - 6x - 12$
C) $y = 2x^3 - 3x$
D) $y = x^3 - 6x + 12$
E) $y = 2x^3 + 3x - 12$

ANTIDERIVATIVES OF TRIG FUNCTIONS & INITIALV VALUE PROBLEMS PRACTICE

1) $\int \left(\frac{1}{t^3} - 2\cos t \right) dt =$

A) $-\frac{3}{t^2} + 2 \sin t + c$

B) $-\frac{1}{2t^2} - 2 \sin t + c$

C) $-\frac{3}{t^2} - 2 \sin t + c$

D) $-\frac{1}{2t^2} + 2 \sin t + c$

E) $-\frac{1}{4t^4} + 2 \sin t + c$

2) $\int \frac{dx}{\csc x} =$

A) -cot x + c
B) -sin x + c
C) cos x + c
D) $\frac{\csc^2 x}{2} + c$
E) -cos x + c

3) $\int \frac{\sin 2\theta}{\cos \theta} d\theta =$

A) -2 cos θ + c
B) cos θ + c
C) cos 2θ + c
D) 2 cos θ + c
E) -cos 2θ + c

4) $\int \csc x \left(\cot x + \sin x \right) dx =$

A) sec x + cos x + c
B) -csc x + c
C) -sec x + tan x + c
D) -csc x + x + c
E) csc x + x + c

5) $\int \tan^2 x \ dx =$

A) tan x - x + c
B) tan x + x + c
C) tan x + c
D) sec x - x + c
E) sec x + x + c

6) If f ′(x) = 2 cos x - 3 sin x and f(0) = 4, then f(x) =

A) 2 sin x + 3 cos x + 1
B) 2 sin x + 3 cos x
C) 2 cos x - 3 sin x + 2
D) 3 sin x - 2 cos x + 6
E) 3 cos x - 2 sin x + 1

7) The derivative of a function f is given by

f ′(x) = $\sec^2 x$ + cos x. If $f\left(\frac{\pi}{4}\right)$ = 1, then f(x) =

A) tan x + sin x + $\frac{\sqrt{2}}{2}$

B) tan x - sin x + $\frac{1}{2}$

C) tan x + sin x + $\frac{1}{2}$

D) tan x + sin x - $\frac{\sqrt{2}}{2}$

E) tan x - sin x - $\frac{\sqrt{2}}{2}$

8) Function f has a second derivative that is given by f ″(x) = x + cos x. Which of the following could be f(x)?

A) $\frac{x^3}{6}$ - 3x + cos x - 1

B) $\frac{x^3}{6}$ + cos x - 1

C) $\frac{x^3}{6}$ - 3x - cos x - 1

D) $\frac{x^3}{6}$ - 3x - sin x - 1

E) $\frac{x^3}{6}$ - 3x + sin x + 1

9) Function g has a second derivative that is given by $g''(x) = x^2 - \sin x$. Which of the following could be function g?

A) $\frac{x^4}{12} + \cos x - x + 1$

B) $\frac{x^4}{12} + \sin x - x$

C) $\frac{x^4}{12} - \cos x + x - 1$

D) $\frac{x^4}{12} - \sin x - x + 1$

E) $\frac{x^4}{4} - \sin x - x + 1$

10) The slope of a curve at each point (x,y) is given by $2 \cos x - x$. Which of the following is an equation of this curve if its graph passes through the point (0,1)?

A) $2 \sin x - x^2 + 1$

B) $\cos^2 x - x^2 + 1$

C) $\cos^2 x - \frac{x^2}{2} + 1$

D) $2 \sin x - \frac{x^2}{2} + 1$

E) $-2 \sin x - \frac{x^2}{2} + 1$

Questions 11 through 17 refer to the following:

Evaluate the given integral.

11) $\int \left(\sin^2 x + \cos^2 x\right) dx$

12) $\int \frac{\sin^3 x - 5}{\sin^2 x} dx$

13) $\int \left(\frac{3}{\cos^2 x} + \sec x \tan x\right) dx$

14) $\int \left(1 - \cos^2 x \sec x\right) dx$

15) $\int \frac{\sin \theta}{\cos^2 \theta} d\theta$

16) $\int \frac{\csc x}{\sin x} dx$

17) $\int \sqrt{x} - \sec^2 x \, dx$

Kuta Software - Infinite Calculus Name____________________

Integration - Logarithmic Rule and Exponentials

Date________________ Period____

Evaluate each indefinite integral.

1) $\int x^{-1}\, dx$

2) $\int 3x^{-1}\, dx$

3) $\int -\frac{1}{x}\, dx$

4) $\int \frac{1}{x}\, dx$

5) $\int -e^{x}\, dx$

6) $\int e^{x}\, dx$

7) $\int 2 \cdot 3^{x}\, dx$

8) $\int 3 \cdot 5^{x}\, dx$

Integration Inverse function

Evaluate each indefinite integral.

1) $\int \frac{1}{\sqrt{16 - x^2}}\, dx$

2) $\int \frac{1}{4 + x^2}\, dx$

3) $\int \frac{1}{x\sqrt{x^2 - 1}}\, dx$

4) $\int \frac{1}{16 + x^2}\, dx$

5) $\int \frac{1}{x\sqrt{x^2 - 4}}\, dx$

6) $\int \frac{1}{\sqrt{25 - x^2}}\, dx$

7) $\int \frac{1}{x\sqrt{x^2 - 81}}\, dx$

8) $\int \frac{1}{4 + x^2}\, dx$